Elementary logic provides the basic rules for constructing sound, compelling arguments. The building blocks of logical arguments are logical statements. A **logical statement** is a declarative sentence which conveys factual information. If the information is correct then we say the statement is **true**; and if the information is incorrect, then we say the statement is **false**.

1. Statements and negations

All logical statements are formed by combining simple statements.

A **simple statement** is a logical statement carrying one piece of information.

Example 1. Each of the following is a simple statement:

(1) The United States Post Office delivers mail.
(2) Paris is the capital of Tanzania.
(3) The polynomial $x^2 - 1$ factors as $(x-1)(x+1)$.

Statements 1 and 3 are true. Statement 2 is false.

Not all short sentences are simple statements

Example 2.

(1) *Close the window.* This is a command. It does not convey information and is not a logical statement.
(2) *The United States Congress is doing an excellent job.* This is a statement of opinion, not a logical statement.

The **negation** of a logical statement is a new logical statement which says the opposite of the original statement. The statement

not p

is true exactly when the original statement p is false. To **negate** a logical statement means to find the statement's negation. Various ways to form the negation of a statement are discussed in the next example.

Example 3. We could negate the statement

> the sky is blue

by forming the statement

> *it is not the case that* the sky is blue.

We can negate any statement in this way, but such constructions are clumsy and sometimes unclear. The negation

> the sky is *not* blue

is much clearer.

A statement containing a negative can often be negated by removing the negative.

Example 4.

(1) The negation of

> loitering is *not* permitted here

is

> loitering is permitted here.

(2) The negation of

> e-mail is *un*reliable

is

> e-mail is reliable.

When a logical statement is used to say something about a collection of objects, the statement must be **quantified**—that is, we must specify which objects in the collection the statement applies to. The basic distinctions between applications are reflected in the words **all** and **some**.

If a logical statement applies to *all* objects in a collection, then it is called a **universally quantified** statement. For example,

(1) *Every* McDonald's serves french fries.
(2) *All* math majors study calculus.

A logical statement which applies to *some* objects in a collection is called an **existentially quantified** statement. For example,

(1) *Some* people attend college.
(2) *There are* people who believe in UFO's.

As illustrated above, an existentially quantified statement asserts that an object of a particular nature exists.

We can contrast universally quantified statements and existentially quantified statements in the following way.

- **Universal quantification**—If every object in the collection fits the description, then the universally qualified statement is true.
- **Existential quantification**—If the existentially quantified statement accurately describes at least one object in the collection, then it is true.

Example 5.

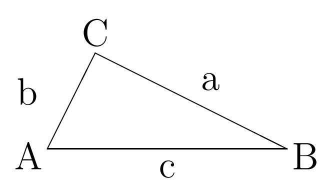
(1) *Some* people are female. This existentially quantified statement applies to the collection of people, and states the existence of a person with the property of being female. It is clearly a true statement.
(2) *All* people are female. This universally quantified statement applied to the collection of people. Since there are people who are not female, it is a false statement.

(3) *Every* whole multiple of four is even. This universally quantified statement applies to the collection of all whole multiples of four. It is true since four is even, and every multiple of an even number is even.

Mathematical knowledge is often expressed with quantified statements:

(1) For all real numbers x and y, $x + y = y + x$.
(2) Some data is normally distributed.
(3) All right triangles obey the Pythagorean Theorem

$$a^2 + b^2 = c^2$$



The negation of a *for all* statement is a *some* statement.

Example 6. The negation of

All birds can fly

is

Some birds can*not* fly.

The negation of a *some* statement is a *for all* statement.

Example 7. The negation of

There exists an honest man

is

All men are *dis*honest.

Example 8. Consider the statement

All numbers can be factored.

A universally quantified statement is false if it does not apply to at least one object in the collection. So the statement is false if some number cannot be factored. We know that 7 cannot be factored (with factors > 1), so the statement is false. The negation of this universally quantified statement is

Some numbers can*not* be factored.

This existentially quantified statement is true.

2. Combining Logical Statements

We combine logical statements to form new logical statements by joining them with an *and* or an *or*.

and

The statement p *and* q is true exactly when both p and q are true.

It is convenient to display this information in the following chart:

The **and** Chart:

p	q	p *and* q
T	T	T
T	F	F
F	T	F
F	F	F

The chart lists the four possible truth values of the two sentences p and q:

(1) both are true;
(2) p is true, but q is false
(3) p is false, but q is true
(4) both are false.

In just the first case is the statement p *and* q true.

or

"Or" is used in two different ways in English:

- *exclusive use:* Tea *or* Coffee?
- *inclusive use:* Cream *or* Sugar?

When you are offered "tea or coffee," it is assumed that you may choose one of the two, *but not both*. On the other hand, when offered "cream or sugar," no eyebrows will be raised if you take them both.

Example 9. Determine whether the following uses of *or* are inclusive or exclusive.

(1) You can kill a vampire by exposing him to sunlight *or* driving a wooden stake through his heart. Suppose you drive a stake through the heart of a vampire at exactly sunrise? Is he still dead?
(2) You buy a car with a 3 year *or* 36,000 mile warranty. If more than 3 years pass *and* you drive the car more than 36,000 miles, is the warranty still expired?
(3) A restaurant offers "soup or salad" with their dinner special. Can you order both?

The first two are examples of the inclusive use or *or*. The third is an example of the exclusive use. You won't get both the soup and the salad without paying an extra charge.

In logic we always use "or" in the inclusive sense. In other words, for us

or means *and/or.*

The **or** Chart:

p	q	p *or* q
T	T	T
T	F	T
F	T	T
F	F	F

The negation of an *and* statement is an *or* statement.

Example 10. The negation of

Sam likes green eggs *and* Sam likes ham

is

Sam does *not* like green eggs *or* Sam does *not* like ham.

The negation of an *or* statement is an *and* statement.

Example 11. The negation of

I am going to the movies on Saturday *or* Sunday

is

I am *not* going on Saturday *and* I am *not* going on Sunday.

More complicated logical statements may be formed by futher combining statements already joined by *and* or *or*.

Example 12. Interpret the following nonpunctuated statement was found on a menu.

All dinners include vegetable *and* soup *or* salad.

Such a statement can be interpreted in two different ways:

(1) You get the vegetable and you must decide between the soup or the salad.
(2) You can must decide between the vegetable and soup combination or the salad.