

## 1. MATH 210 FINITE MATHEMATICS

- Chapter 1.1 Area, Pythagorean Theorem, Circles
- Chapter 1.2 Lines and Slope
- Chapter 1.4 Intesection of Two Lines
- Chapter 2.1 Systems of Equations
- Professor Richard Blecksmith
- Dept. of Mathematical Sciences
- Northern Illinois University

## 2. CHAPTER 1 HOMEWORK

Note the following changes in the Homework!

- Chapter 1.1, page 6: 23, 28
- Chapter 1.2, page 18: 7, 13, 42, 46, 62, 64
- Chapter 1.4, page 46: 1, 4, 6
- Chapter 2.7: Input–Output Model Deleted

## 3. HOW BIG IS AN ACRE?

Which is bigger: an acre or a football field?

Webster Unabridged Dictionary: An acre is a U. S. and English measurement, meant to represent the amount of land a yoke of oxen could plow in one day. **That's helpful!**

One acre equals 160 square rods **That clears it right up!**

What's a rod?

By Webster again, one rod equals 5.5 yards.

So 1 square rod =  $5.5^2$  square yards  
= 30.25 square yards

1 acre =  $160 \times 30.25$  square yards = 4840 square yards

## 4. FOOTBALL FIELD VERSUS AN ACRE

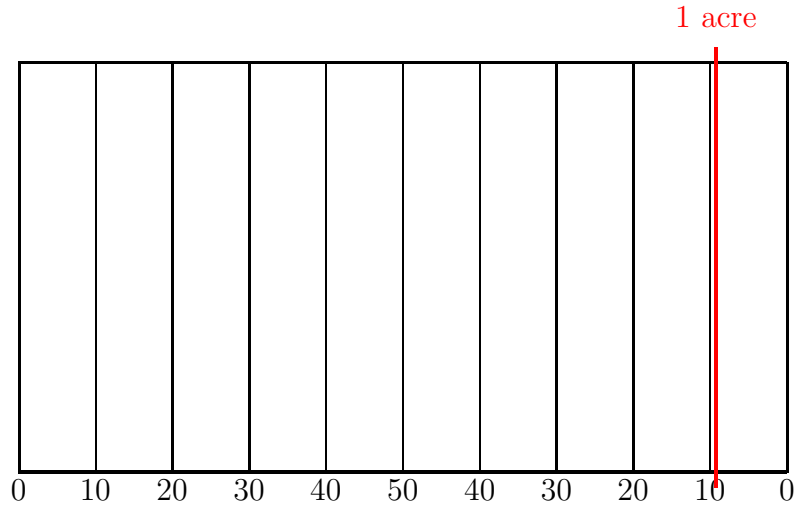
A collegiate football field measures 100 by  $53\frac{1}{3}$  yards.

$= 5333\frac{1}{3}$  square yards

1 acre =  $160 \times 30.25$  square yards = 4840 square yards

So a football field is about 10% bigger than an acre.

### 5. FOOTBALL FIELD VERSUS AN ACRE



Go Huskies!

### 6. THE PYTHAGOREAN THEOREM

- An angle of 90 degrees is called a **right angle**.
- A **right triangle** is a triangle where one angle is a right angle.
- The side opposite the right angle is called the **hypotenuse**.
- The other two sides are called the **legs** of the right triangle.

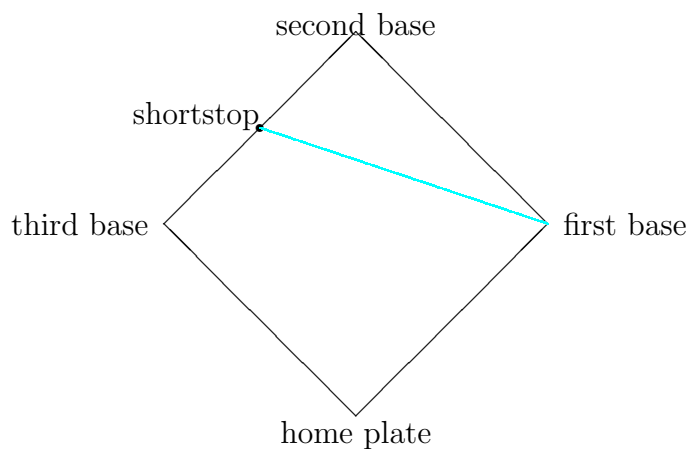
If the legs of a right triangle measure  $a$  and  $b$ , and the hypotenuse has length  $c$ , then

$$a^2 + b^2 = c^2$$

Common Examples: 3 – 4 – 5    5 – 12 – 13

### 7. BASEBALL DIAMOND

A baseball diamond is a square, 90 feet between bases. The shortstop is playing exactly halfway between second base and third. How far is the shortstop from first base?



### 8. BASEBALL DIAMOND

Solution: Use the Pythagorean Theorem with side lengths  $a = 90$  and  $b = 45$ :

$$c^2 = a^2 + b^2$$

$$c^2 = 90^2 + 45^2$$

$$c^2 = 8100 + 2025 = 10125$$

$$c = \sqrt{10125} = 100.6 \text{ feet}$$

### 9. DISTANCE FORMULA

The distance between two points  $P = (x_1, y_1)$  and  $Q = (x_2, y_2)$  is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Find the distance between  $(1, 3)$  and  $(4, 7)$

Solution: By the distance formula

$$d = \sqrt{(4 - 1)^2 + (7 - 3)^2} = \sqrt{3^2 + 4^2} = 5$$

## 10. CIRCLES

The important theorems:

$$\boxed{\text{Circumference} = 2\pi \times \text{radius}}$$

$$\boxed{\text{Area} = \pi \times \text{radius squared}}$$

Problem. What is a better deal: a 10 inch pizza for \$7 or a 14 inch pizza for \$10?

Solution: The radius of the 10 inch pizza is  $r = 5$ .

So the area of the small pizza is

$$A_{\text{small}} = \pi \cdot 5^2 = 25\pi$$

The radius of the 14 inch pizza is  $r = 7$ .

So the area of the large pizza is

$$A_{\text{large}} = \pi \cdot 7^2 = 49\pi$$

The large pizza has almost twice the area, but only costs  $10/7 = 1.4$  times as much.

The large pizza is a much better deal.

## 11. SLOPE

**Problem:** How to find the slope of a line:

If you know two points

$$P = (x_1, y_1) \quad \text{and} \quad Q = (x_2, y_2)$$

Subtract:  $y_2 - y_1$

Subtract:  $x_2 - x_1$

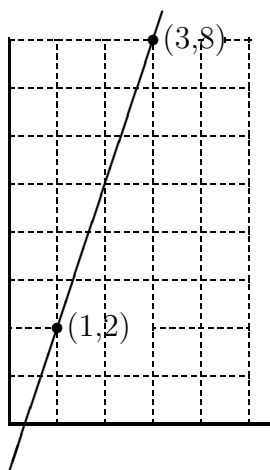
Now divide:

$$\text{slope} = \boxed{\frac{y_2 - y_1}{x_2 - x_1}}$$

## 12. EXAMPLE

Find the slope of the line through the points

$$P = (1, 2) \quad \text{and} \quad Q = (3, 8)$$



$$\begin{aligned} \text{slope} &= \frac{8-2}{3-1} \\ &= \frac{6}{2} = 3 \end{aligned}$$

## 13. FINDING SLOPE FROM EQUATION

Suppose you have an equation for the line:

**Case 1.** If the equation looks like

$$y = mx + b$$

then  $m$  is the slope.

**Examples:**

1. The slope of  $y = 2x + 3$  is 2.
2. The slope of  $y = -\frac{3}{4}x + \frac{1}{2}$  is  $-\frac{3}{4}$ .

## 14. CASE 2

**Case 2.** If the equation looks like

$$ax + by = c$$

Solve for  $y$  to find the slope.

**Example:**

To find the slope of the line  $2x + 3y = 6$

Solve  $3y = -2x + 6$  or  $y = \frac{-2x+6}{3} = -\frac{2}{3}x + 2$

The slope is  $-\frac{2}{3}$

## 15. GRAPHING LINES: CASE 1

Equation  $y = mx + b$

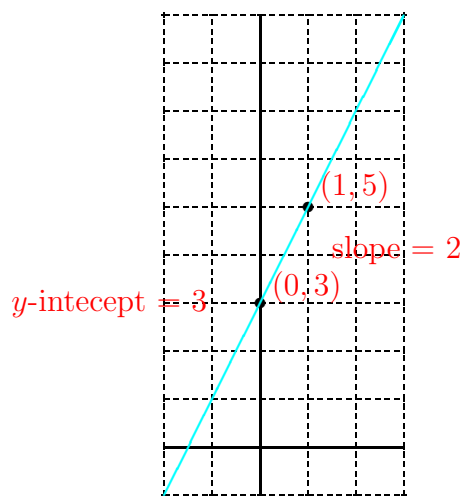
$m$  is the slope

$b$  is the y-intercept

The line goes through the point  $(0, b)$  with slope  $m$ .

This means that if  $x$  increases by 1, then  $y$  increases by  $m$  if  $m$  is positive and decreases by  $m$  if  $m$  is negative.

These two values are enough to draw the line.

16. GRAPH  $y = 2x + 3$ 

## 17. GRAPHING LINES: CASE 2

Equation  $ax + by = c$

Find the  $y$ -intercept by plugging in  $x = 0$ .

Darken the  $y$ -intercept  $(0, b)$  on the  $y$ -axis

Find the  $x$ -intercept by plugging in  $y = 0$ .

Darken the  $x$ -intercept  $(a, 0)$  on the  $x$ -axis.

The line goes through the point  $(a, 0)$  and  $(0, b)$ .

18. GRAPH  $2x + 3y = 6$ 

Find the  $y$ -intercept by plugging in  $x = 0$ .

$$2 \cdot 0 + 3y = 6$$

$$0 + 3y = 6$$

$$3y = 6$$

Divide by 3:

$$y = 2$$

$y$ -intercept is  $(0, 2)$

Find the  $x$ -intercept by plugging in  $y = 0$ .

$$2x + 3 \cdot 0 = 6$$

$$2x + 0 = 6$$

$$2x = 6$$

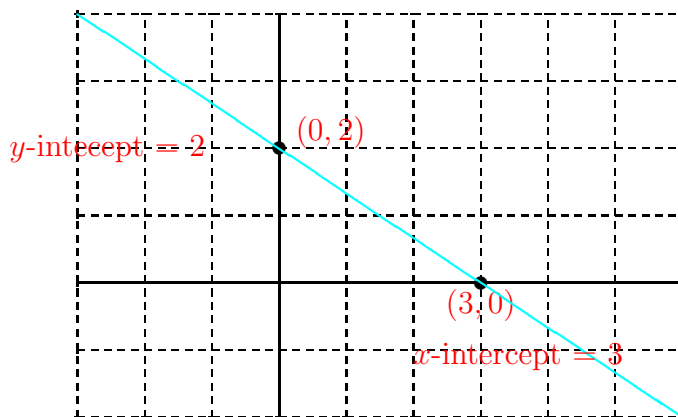
Divide by 2:

$$x = 3$$

$x$ -intercept is  $(3, 0)$

19. GRAPH  $2x + 3y = 6$ 

$y$ -intercept is  $(0,2)$   $x$ -intercept is  $(3,0)$



## 20. CONNECTION TO GEOMETRY

Given two lines.

- If they have different slopes, then they intersect.
- If they have the same slope, then they are parallel.
- Unless they are the same line.

## 21. INTERSECT, PARALLEL, SAME?

Line 1:  $3x + 5y = 15$

Line 2:  $x - 3y = 9$

Line 1 can be rewritten  $5y = -3x + 15$

Dividing by 5:  $y = -\frac{3}{5}x + 3$

Line 2 can be rewritten

$-3y = -x + 9$

Dividing by  $-3$ :

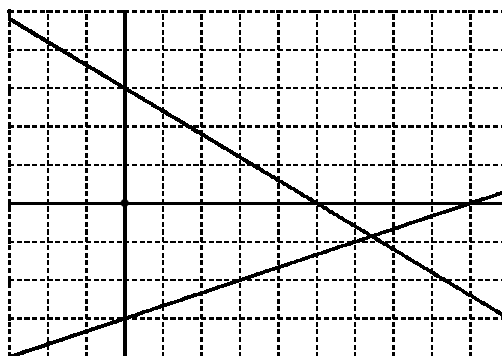
$y = \frac{-1}{-3}x + \frac{9}{-3}$

$$y = \frac{1}{3}x - 3$$

Since two lines have different slopes, they must intersect.

## 22. PICTURE

Line 1



Line 2

## 23. INTERSECT, PARALLEL, SAME?

Line 1:  $x + 2y = 4$

Line 2:  $12 - 3x = 6y$

Line 1 can be rewritten  $2y = -x + 4$

Dividing by 2:  $y = -\frac{1}{2}x + 2$

Line 2 can be rewritten

$$6y = -3x + 12$$

Dividing by 6:

$$y = -\frac{1}{2}x + 2$$

They are the same line.

## 24. INTERSECT, PARALLEL, SAME?

Line 1:  $y = x - 1$

Line 2:  $4x - 4y = 6$

Line 1 has slope 1.

Line 2 can be rewritten

$$-4y = -4x + 6$$

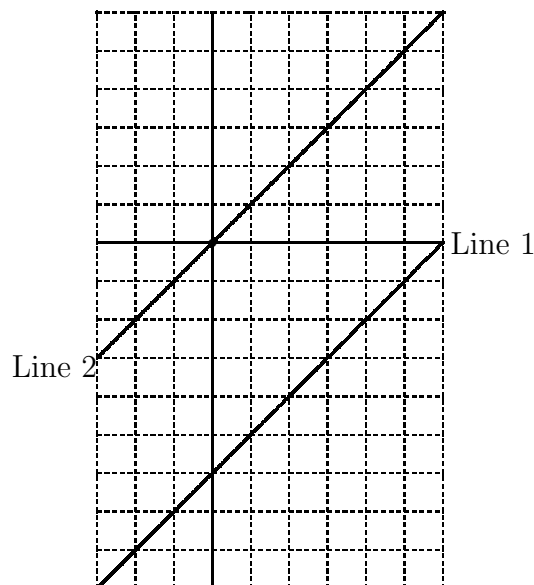
Dividing by  $-4$ :

$$y = \frac{-4x+6}{-4} = \frac{-4x}{-4} + \frac{6}{-4} = x - \frac{3}{2}$$

Since Line 2 also has slope 1, it is either parallel to Line 1 or the same.

Since the  $y$ -intercept of Line 1 is  $-1$  and the  $y$ -intercept of Line 2 is  $-3/2$ , the lines are parallel.

#### 25. PICTURE



#### 26. LINEAR VERSUS NONLINEAR

Many functions are linear, that is, their graphs are straight lines.

Example: You work at an hourly wage of \$6.50.

Then your weekly salary is linear, as a function of the number of hours worked:

$$\text{Salary} = 6.5 \times \text{hours worked}$$

On your first hour, you make \$6.50.

On your second hour, you make \$6.50.

On your 19th hour, you make \$6.50.

## 27. NOT ALL FUNCTIONS ARE LINEAR

Example: Road & Track Magazine tested Chrysler's PT Cruiser.

They needed 131 feet to stop the car from a speed of 60 mph and 232 feet to stop from a speed of 80 mph.

Increasing the speed by 20 mph almost doubles the distance to stop.

This is not linear behavior.

Sir Issac Newton: inventor of Calculus (and a popular fig-filled cookie):

Stopping distance increases with the square of the speed.

$$\boxed{\text{Stopping Distance} = k \times \text{Speed}^2}$$

where  $K$  is the (braking) constant.

## 28. FINDING BRAKING CONSTANT

Plugging in Distance = 131 and Speed = 60 allows us to solve for  $K$ :

$$131 = k \cdot 60^2$$

Solve for  $k$  by dividing by  $60^2$ :

$$k = \frac{131}{60^2} = .0364$$

$$\boxed{\text{Stopping Distance} = .0364 \times \text{Speed}^2}$$

## 29. TESTING NEWTON

This equation predicts that when Speed = 80, the stopping distance will be

$$\text{Stopping Distance} = .0364 \times 80^2 = 232.9 \text{ feet}$$

Compare this with the actual value found by Road & Track's test drivers:

Stopping distance at 80 mph was 232 feet.

Way to go, Newton.

Not bad for a guy who predated the automobile by 200 years.

## 30. STOPPING DISTANCES

Speed	Stopping Dist	Speed	Stopping Dist
5	.9	55	110.1
10	3.6	60	131
15	8.2	65	153.7
20	14.6	70	178.3
25	22.7	75	204.7
30	32.8	80	232.9
35	44.6	85	262.9
40	58.2	90	294.8
45	73.7	95	328.4
50	91.0	100	363.9

## 31. MORAL

If you increase your speed by 5 mph, it requires an additional 4.6 feet to stop if you are going 10 to 15 mph.

It requires an additional 30 feet to stop if you are going 80 to 85 mph.

Moral: The faster you drive, the more distance you should leave for the car in front of you to stop.

This is not a linear function.

## 32. WHERE DO TWO LINES CROSS?

Determine where the following two lines intersect:

$$\begin{cases} 3x + 5y = 15 \\ x - 3y = 9 \end{cases}$$

- The Addition–Subtraction Method
- Multiply the two equations by appropriate numbers so that
- When they are added (or subtracted)
- One of the variables drops out of the equation.

## 33. INTERSECTION PROBLEM REVISITED

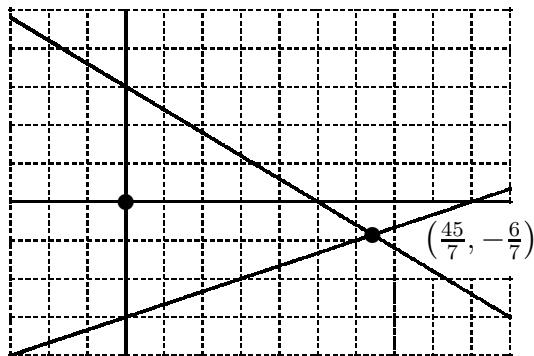
- Step 1. Write the equations.
  - (1)  $3x + 5y = 15$
  - (2)  $x - 3y = 9$
- Step 2. Multiply so that  $x$  drops out
  - (1)  $3x + 5y = 15$
  - $3 \times$  (2)  $3x - 9y = 27$
  - Subtract  $0x + 14y = -12$
- So  $y = -\frac{12}{14} = -\frac{6}{7}$

## 34. INTERSECTION PROBLEM CONT'D

- Step 3. Plug back into original equation (2)
  - $-x - 3y = 9$
  - $-x - 3\left(-\frac{6}{7}\right) = 9$
  - $-x + \frac{18}{7} = 9$
  - $-x = 9 - \frac{18}{7} = \frac{63}{7} - \frac{18}{7} = \frac{45}{7}$
- Step 4. Write down the solution:
  - $x = \frac{45}{7}, y = -\frac{6}{7}$ ,

## 35. PICTURE

Line 1



Line 2

## 36. PEACH PROBLEM

Brenda decides to sell the peaches from her backyard to the women in the neighborhood. She sells Mrs. Jones 10 large peaches and 15 small peaches for \$9.50. She sells Mrs. Williams 15 large peaches and 10 small peaches for \$10.50. How much does she charge for a small and a large peach?

## 37. PEACHES TO MATH

- Step 1. Make the variables.
  - $L$  = the price of a large peach
  - $S$  = the price of a small peach
- Step 2. Write the equations.
  - (1)  $10L + 15S = 9.5$
  - (2)  $15L + 10S = 10.5$
- Step 3. Solve for one of the variables.
  - $3 \times (1)$   $30L + 45S = 28.5$
  - $2 \times (2)$   $30L + 20S = 21$
  - Subtract  $0L + 25S = 7.5$
- So  $S = \frac{7.5}{25} = .30$

## 38. PEACH SOLUTION

- Step 4. Plug back into original equation (1)
  - $10L + 15 \times .30 = 9.50$
  - $10L + 4.50 = 9.50$

$$- 10L = 5.00$$

$$- L = .50$$

- Step 5. Write down the solution:
- A large peach costs 50 cents; a small peach costs 30 cents.

### 39. THREE EQUATIONS

Solve for  $x$ ,  $y$ ,  $z$ :

$$3x + 6y - 5z = 0$$

$$2x + 4y - 3z = 1$$

$$x + y + 2z = 9$$

To solve a system of three equations in three variables, we work systematically.

First, eliminate the variable  $x$  from the first two equations.

Then eliminate  $x$  from equations one and three.

We now have two equations in the two variables  $y$  and  $z$ , which we know how to solve.

### 40. THREE EQUATIONS—A START

$$(1): \quad 3x + 6y - 5z = 0$$

$$(2): \quad 2x + 4y - 3z = 1$$

$$(3): \quad x + y + 2z = 9$$

To eliminate  $x$  from equations (1) and (2) we have to multiply equation (1) by 2 and equation (2) by  $-3$ .

A better way: because the coefficient of  $x$  in equation (3) is a  $\boxed{1}$ , it would be easier if equation (3) were the first equation.

Well, it doesn't matter what order we write the equations. So we can reorder them any way we please.

So, voila:

$$(1): \quad x + y + 2z = 9$$

$$(2): \quad 2x + 4y - 3z = 1$$

$$(3): \quad 3x + 6y - 5z = 0$$

#### 41. EQUATIONS (1) AND (2)

$$(1): \quad x + y + 2z = 9$$

$$(2): \quad 2x + 4y - 3z = 1$$

Step 2. Multiply so that  $x$  drops out

$$2 \times (1) \quad 2x + 2y + 4z = 18$$

$$(2) \quad 2x + 4y - 3z = 1$$

$$\text{Subtract} \quad 0x - 2y + 7z = 17$$

Set this equation aside, we will need it later.

#### 42. EQUATIONS (1) AND (3)

$$(1): \quad x + y + 2z = 9$$

$$(3): \quad 3x + 6y - 5z = 0$$

Step 2. Multiply so that  $x$  drops out

$$3 \times (1) \quad 3x + 3y + 6z = 27$$

$$(3) \quad 3x + 6y - 5z = 0$$

$$\text{Subtract} \quad 0x - 3y + 11z = 27$$

Set this equation aside, we will need it later.

#### 43. THE $y$ - $z$ EQUATIONS

$$(1'): \quad -2y + 7z = 17$$

$$(2'): \quad -3y + 11z = 27$$

Step 2. Multiply so that  $x$  drops out

$$-3 \times (1') \quad 6y - 21z = -51$$

$$2 \times (2') \quad -6y + 22z = 54$$

$$\text{Add} \quad 0y + 1z = 3$$

Our first piece of the puzzle:  $z = 3$

#### 44. FINDING $y$

Substitute  $z = 3$  into

$$(1'): \quad -2y + 7z = 17$$

$$-2y + 7(3) = 17$$

$$-2y + 21 = 17$$

$$-2y = -4$$

$$y = 2$$

Our second piece of the puzzle:  $y = 2$

#### 45. FINDING $x$

Substitute  $y = 2$  and  $z = 3$  into

$$(1): \quad x + y + 2z = 9$$

$$x + 2 + 2(3) = 9$$

$$x + 8 = 9$$

$$x = 1$$

The final piece of the puzzle.

We now know all three variables:

$$x = 1, y = 2, \text{ and } z = 3$$