

MATH 210 FINITE MATHEMATICS

Chapter 3.2 Linear Programming Problems

Chapter 3.3 Graphical Solution

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Math 210 Website: <http://math.niu.edu/courses/math210>

1. NUTRITION PROBLEM

Sarah decides to make rice and soybeans part of her staple diet.

The object is to design a lowest-cost diet that provides certain minimum levels of protein, calories, and vitamin B_{12} (riboflavin).

One cup of uncooked rice costs 21 cents and contains 15 grams of protein, 810 calories, and $\frac{1}{9}$ mg of riboflavin.

One cup of uncooked soybeans costs 14 cents and contains 22.5 grams of protein, 270 calories, and $\frac{1}{3}$ mg of riboflavin.

The minimum daily requirements are: 90 grams of protein, 1620 calories, and 1 mg of B_{12} .

Design the lowest cost diet that meets these requirements.

3. ORGANIZING THE DATA

Category	Rice	Soybeans	Requirement
Protein	15	22.5	90
Calories	810	270	1620
Riboflavin	$\frac{1}{9}$	$\frac{1}{3}$	1
Cost	21	14	C

Nutrition Inequalities:

- **Protein** $15x + 22.5y \geq 90$
- **Calories** $810x + 270y \geq 1620$
- **Riboflavin** $\frac{1}{9}x + \frac{1}{3}y \geq 1$

Cost Equation:

- **Cost** $C = 21x + 14y$

2. SET THE VARIABLES

Let

- x = the number of cups of uncooked rice in her diet
- y = the number of cups of uncooked soybeans in her diet

The problem is to find the values of x and y which will

- minimize the cost and
- provide the daily requirements of protein, calories, and riboflavin.

4. SIMPLIFY THE INEQUALITIES

Nutrition Inequalities:

(1) Protein: $15x + 22.5y \geq 90$

(2) Calories: $810x + 270y \geq 1620$

(3) Riboflavin: $\frac{1}{9}x + \frac{1}{3}y \geq 1$

To remove the decimal 22.5 in (1), multiply (1) by 2:

(1') Protein: $30x + 45y \geq 180$

Now notice that 30, 45, and 180 are all divisible by 15. So divide (1') by 15:

(1'') Protein: $2x + 3y \geq 12$

In (2) notice that 810, 270, and 1620 are all divisible by 270. So divide (2) by 270:

(2') Calories: $3x + y \geq 6$

To remove the denominators in (3), multiply (3) by 9:

(3') Riboflavin: $x + 3y \geq 9$

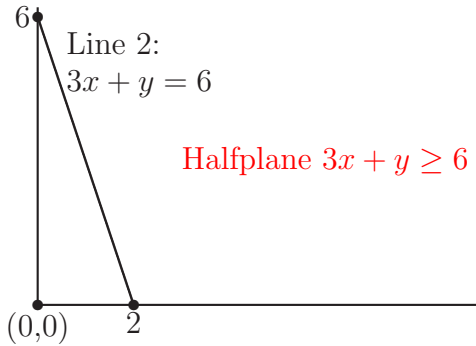
5. NUTRITION WORDS \rightarrow MATH

Minimize the cost $C = 21x + 14y$
subject to the constraints

- $2x + 3y \geq 12$
- $3x + y \geq 6$
- $x + 3y \geq 9$
- $x \geq 0, y \geq 0$

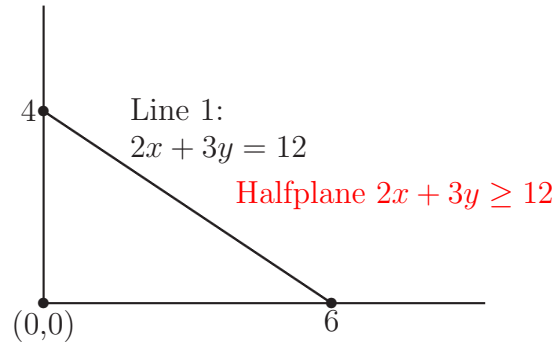
7. CALORIE HALFPLANE

Halfplane: $3x + y \geq 6$
Line: $3x + y = 6$
 x -intercept: $(2, 0)$
 y -intercept: $(0, 6)$
Test point equation: $0 \geq 6$
Position: above line



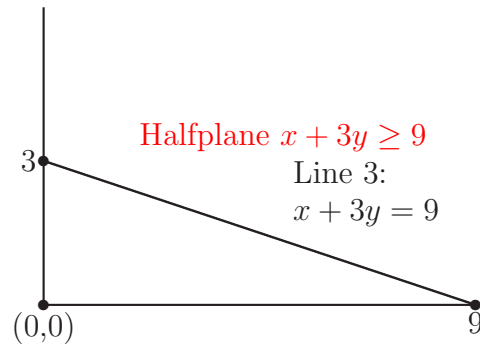
6. PROTEIN HALFPLANE

Halfplane: $2x + 3y \geq 12$
Line: $2x + 3y = 12$
 x -intercept: $(6, 0)$
 y -intercept: $(0, 4)$
Test point equation: $0 \geq 12$
Position: above line



8. RIBOFLAVIN HALFPLANE

Halfplane: $x + 3y \geq 9$
Line: $x + 3y = 9$
 x -intercept: $(9, 0)$
 y -intercept: $(0, 3)$
Test point equation: $0 \geq 9$
Position: above line



9. WHERE DO PROTEIN AND CALORIE LINES INTERSECT?

- Write the equations:
Protein $2x + 3y = 12$
Calorie $3x + y = 6$
- Multiply Calorie equation by 3
Calorie $9x + 3y = 18$
Protein $2x + 3y = 12$
Subtract $7x = 6$
- So $x = \frac{6}{7}$
- Plug $x = \frac{6}{7}$ back into the Calorie equation
 $y = 6 - 3x = 6 - 3 \cdot \frac{6}{7} = \frac{42}{7} - \frac{18}{7} = \frac{24}{7}$
- The intersection point is $(\frac{6}{7}, \frac{24}{7})$

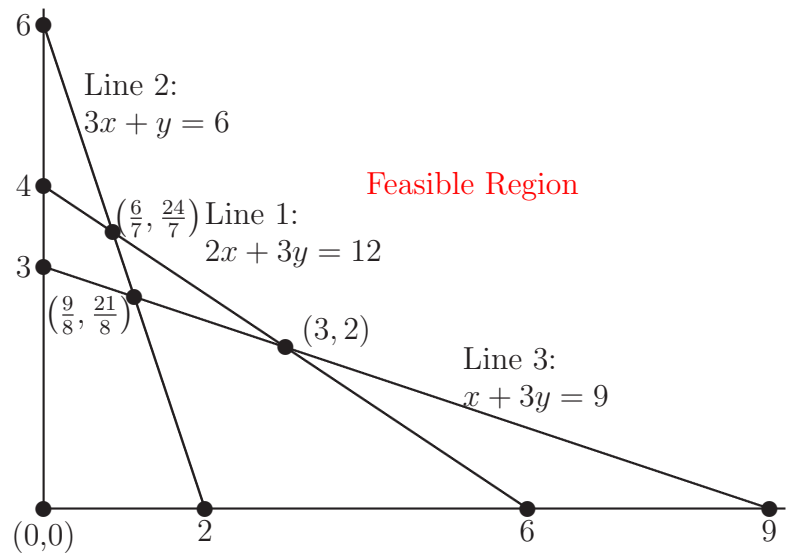
11. WHERE DO CALORIE AND RIBOFLAVIN LINES INTERSECT?

- Write the equations:
Calorie $3x + y = 6$
Ribofl $x + 3y = 9$
- Multiply Calorie equation by 3
Calorie $9x + 3y = 18$
Ribofl $x + 3y = 9$
Subtract $8x = 9$
- So $x = \frac{9}{8}$
- Plug $x = \frac{9}{8}$ back into the Calorie equation
 $y = 6 - 3x = 6 - 3 \cdot \frac{9}{8} = \frac{48}{8} - \frac{27}{8} = \frac{21}{8}$
- The intersection point is $(\frac{9}{8}, \frac{21}{8})$

10. WHERE DO PROTEIN AND RIBOFLAVIN LINES INTERSECT?

- Write the equations:
Protein $2x + 3y = 12$
Ribofl $x + 3y = 9$
Subtract $x = 3$
- Plug $x = 3$ back into the Riboflavin equation
 $y = \frac{9 - x}{3} = \frac{9 - 3}{3} = 2$
- The intersection point is $(3, 2)$

12. FEASIBLE REGION



13. SOLVING THE NUTRITION PROBLEM

The list of corner points is:

$(0, 6)$, $(\frac{6}{7}, \frac{24}{7})$, $(3, 2)$, $(9, 0)$

The intersection $(\frac{9}{8}, \frac{21}{8})$ of Lines 2 and 3 is not a corner point because it is not on halfplane

$2x + 3y \geq 12$:

$$2 \cdot \frac{9}{8} + 3 \cdot \frac{21}{8} = \frac{18}{8} + \frac{63}{8} = \frac{81}{8} = 10.125 < 12$$

x	y	$C = 21x + 14y$
0	6	84
$\frac{6}{7}$	$\frac{24}{7}$	$18 + 48 = 66$
3	2	91
9	0	189

Minimum value of C is 66 at the point $(\frac{6}{7}, \frac{24}{7})$

15. SOLUTION: DETERMINE VARIABLES

At first it appears that 4 variables will be needed, since there are two stores and two warehouses, hence 4 possible shipping combinations.

A closer look shows that only two variables x and y are needed.

For if x represents the number of TV sets to be shipped from DeKalb to Schaumburg, then since Schaumburg needs 25 sets, the number of TV sets to be shipped from Elkhart to Schaumburg is $25 - x$.

Similarly, if y represents the number of TV sets to be shipped from DeKalb to Aurora, then since Aurora needs 30 sets, the number of TV sets to be shipped from Elkhart to Aurora is $30 - y$.

14. TRANSPORTATION PROBLEM

A Chicago TV dealer has stores in Schaumburg and Aurora and warehouses in DeKalb and Elkhart, Indiana. The cost of shipping a 48 inch flat screen TV set from DeKalb to Schaumburg is \$6; from DeKalb to Aurora is \$3; from Elkhart to Schaumburg, \$9; and from Elkhart to Aurora is \$5. Suppose the Schaumburg store orders 25 TV sets and the Aurora store orders 30. The DeKalb warehouse has a stock of 45 TV sets and the Elkhart warehouse has 40.

What is the most economical way to supply the required sets to the two stores?

16. COST TABLE

Warehouse–Store	Number	Cost per set
DeKalb–Schaumburg	x	\$6
DeKalb–Aurora	y	\$3
Elkhart–Schaumburg	$25 - x$	\$9
Elkhart–Aurora	$30 - y$	\$5

Total Cost:

$$\begin{aligned} C &= 6x + 3y + 9(25 - x) + 5(30 - y) \\ &= 6x + 3y + 225 - 9x + 150 - 5y \\ &= -3x - 2y + 375 \end{aligned}$$

17. CONSTRAINTS

There are two kinds of constraints:

- none of x , y , $25 - x$, $30 - y$ can be negative
Equivalently, $x \geq 0$, $y \geq 0$, $x \leq 25$, $y \leq 30$
- a warehouse cannot ship more TVs than it has in stock.

Since DeKalb ships $x + y$ sets and has 45 in stock, we get the constraint:
 $x + y \leq 45$

Since Elkhart ships $(25 - x) + (30 - y)$ sets and has 40 in stock, we get the constraint: $(25 - x) + (30 - y) \leq 40$
or $55 - x - y \leq 40$
or $15 \leq x + y$

19. THE HALFPLANES

Halfplane	Line	Intercept(s)	Position
$15 \leq x + y$	$15 = x + y$	(15, 0) (0, 15)	above
$x + y \leq 45$	$x + y = 45$	(45, 0) (0, 45)	below
$y \leq 30$	$y = 30$	(0, 30)	below
$x \leq 25$	$x = 25$	(25, 0)	left

The constraints $x, y \geq 0$ imply that the region lies in the First Quadrant.

21. SOLVING THE TRANSPORTATION PROBLEM

The list of corner points is:
(0, 15), (0, 30), (15, 30), (25, 20), (25, 0) and (15, 0)

x	y	$C = 375 - 3x - 2y$
0	15	345
0	30	315
15	30	270
25	20	260
25	0	300
15	0	330

Minimum value of C is

- 260 and occurs at the point (25, 20)

18. TRANSPORTATION WORDS \rightarrow MATH

Minimize the cost $C = -3x - 2y + 375$
subject to the constraints

- $x \leq 25$, $y \leq 30$
- $15 \leq x + y$
- $x + y \leq 45$
- $x \geq 0$, $y \geq 0$

20. FEASIBLE REGION

