1. Math 210 Finite Mathematics

- Chapter 4.2 Linear Programming Problems
  The Simplex Method
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2. The Simplex Method

- For each inequality introduce a slack variable to convert the inequality into an equation.
- Write each slack equation as a row in the Simplex Tableu (French for "matrix")
- Write the function to be maximized as the bottom row of the Simplex Tableau
- Find the Pivot Point (row and column)
- Pivot and check the tableau to see if you are done.

3. Step 1: Slack Variables

This is easier to illustrate that to explain.

Consider the inequality:

$$x + y \leq 16$$

Rewrite this inequality as the equation:

$$x + y + u = 16$$

Note that

- $u = 16 - x - y$
- $x + y \leq 16$ is equivalent to $0 \leq 16 - x - y$
  which is equivalent to $u \geq 0$

4. Step 1: Slack Variables

Each inequality in your systems requires its own slack variable.

The system of inequalities

(1) $x + y \leq 16$
(2) $5x + 2y \leq 50$
(3) $y \leq 12$

can be rewritten as

(1) $x + y + u = 16$
(2) $5x + 2y + v = 50$
(3) $y + w = 12$

5. Step 2: The Simplex Tableau

The Simplex Tableau has a row for each slack equation in the system, plus an additonal row at the bottom for the objective function.

There is a column for each regular variable, for each slack variable, and for the objective function variable (usually Profit or Cost).

The last column holds the constants to the right of the equal sign for each equation.

6. Simplex Tableau Illustration

The first three rows of the Simplex Tableau for

$x + y + u = 16$

$5x + 2y + v = 50$

$y + w = 12$

are

x	y	u	v	w	P	const
1	1	1	0	0	0	16
5	2	0	1	0	0	50
0	1	0	0	1	0	12

7. Rewriting the Objective Function

Suppose the objective function is

$$P = 7x + 5y$$

We rewrite this as

$$-7x + -5y + P = 0$$

and add this equation as the bottom row of the Simplex Tableau

x	y	u	v	w	P	const
1	1	1	0	0	0	16
5	2	0	1	0	0	50
0	1	0	0	1	0	12
-7	-5	0	0	0	1	0

8. Basic and Non-basic Variables

Notice that in the Simplex Tableau

x	y	u	v	w	P	const
1	1	1	0	0	0	16
5	2	0	1	0	0	50
0	1	0	0	1	0	12
-7	-5	0	0	0	1	0

the columns under the variables u, v, w, and P are unit columns, meaning the column consists of a single 1 and the other entries are zero.

The variables corresponding to unit columns—u, v, w, and P—are called basic variables; the other variables—x and y are called non-basic variables.

9. Basic and Non-basic Variables

x	y	u	v	w	P	const
1	1	1	0	0	0	16
5	2	0	1	0	0	50
0	1	0	0	1	0	12
-7	-5	0	0	0	1	0

Basic variables: u, v, w, and P

Non-basic: x and y

If we set the non-basic variables to zero, then it is easy to read the values of the basic variables:

$u = 16$

$v = 50$

$w = 12$

$P = 0$

10. Finding the Pivot Column

Examine the last row of the Simplex Tableau, not counting the last entry in the constant column.

If all the entries are ≥ 0, stop.

You have found a solution.

If some of the entries in the bottom row are negative,

find the column with the negative entry which is largest in absolute value.

This is the pivot column.

If there is a tie for the most negative entry, flip a coin, and choose one of them.

11. Pivot Column Example

In the Simplex Tableau

x	y	u	v	w	P	const
1	1	1	0	0	0	16
5	2	0	1	0	0	50
0	1	0	0	1	0	12
-7	-5	0	0	0	1	0

The entries in the bottom row, not counting the constant column, are highlighted in red.

The most negative entry is -7 and occurs in column 1.

So the pivot column is 1.

12. Finding the Pivot Row

Divide each entry in the constant column by the corresponding entry in the pivot column.

Record this ratio to the right of that row.

Two rules:

(i) the entry in the pivot column must be positive

(ii) do not include the bottow row.

The row with the smallest ratio is the pivot row.

13. Pivot Row Example

In the Simplex Tableau

x	y	u	v	w	P	const	ratio
1	1	1	0	0	0	16	$\frac{16}{1} = 16$
5	2	0	1	0	0	50	$\frac{50}{5} = 10$
0	1	0	0	1	0	12	none
-7	-5	0	0	0	1	0	

The smallest ratio is 10 and occurs in row 2.

So the pivot row is 2.

Note that we do not compute a ratio for the third row because the entry 0 in row 3, col 1 is not positive.

14. Pivot at row 2, col 1

1	1	1	0	0	0	16
5	2	0	1	0	0	50
0	1	0	0	1	0	12
−7	−5	0	0	0	1	0

1	1	1	0	0	0	16
5	2	0	1	0	0	50
0	1	0	0	1	0	12
−7	−5	0	0	0	1	0

Pivot at row 2 col 1

1	1	1	0	0	0	16
5	2	0	1	0	0	50
0	1	0	0	1	0	12
-7	-5	0	0	0	1	0

Multiply row 2 by $\frac{1}{5}$

1	1	1	0	0	0	16
1	$\frac{2}{5}$	0	$\frac{1}{5}$	0	0	10
0	1	0	0	1	0	12
-7	-5	0	0	0	1	0

1	1	1	0	0	0	16
1	$\frac{2}{5}$	0	$\frac{1}{5}$	0	0	10
0	1	0	0	1	0	12
-7	-5	0	0	0	1	0

Add $-1 \times$ row 2 to row 1

$-1 \times 1 + 1 = 0$

$-1 \times \frac{2}{5} + 1 = \frac{3}{5}$

$-1 \times \frac{1}{5} + 0 = -\frac{1}{5}$

$-1 \times 10 + 16 = 6$

0	$\frac{3}{5}$	1	$-\frac{1}{5}$	0	0	6
1	$\frac{2}{5}$	0	$\frac{1}{5}$	0	0	10
0	1	0	0	1	0	12
-7	-5	0	0	0	1	0

0	$\frac{3}{5}$	1	$-\frac{1}{5}$	0	0	6
1	$\frac{2}{5}$	0	$\frac{1}{5}$	0	0	10
0	1	0	0	1	0	12
-7	-5	0	0	0	1	0

Add $0 \times$ row 2 to row 3

$0 \times 1 + 0 = 0$

$0 \times \frac{2}{5} + 1 = 1$

$0 \times \frac{1}{5} + 0 = 0$

$0 \times 10 + 12 = 12$

0	$\frac{3}{5}$	1	$-\frac{1}{5}$	0	0	6
1	$\frac{2}{5}$	0	$\frac{1}{5}$	0	0	10
0	1	0	0	1	0	12
-7	-5	0	0	0	1	0

0	$\frac{3}{5}$	1	$-\frac{1}{5}$	0	0	6
1	$\frac{2}{5}$	0	$\frac{1}{5}$	0	0	10
0	1	0	0	1	0	12
−7	−5	0	0	0	1	0

Add 7 × row 2 to row 4

$$7 \times 1 + -7 = 0$$

$$7 \times \frac{2}{5} + -5 = -\frac{11}{5}$$

$$7 \times \frac{1}{5} + 0 = \frac{7}{5}$$

$$7 \times 10 + 0 = 70$$

0	$\frac{3}{5}$	1	$-\frac{1}{5}$	0	0	6
1	$\frac{2}{5}$	0	$\frac{1}{5}$	0	0	10
0	1	0	0	1	0	12
0	$-\frac{11}{5}$	0	$\frac{7}{5}$	0	1	70

15. The Second Tableau

After pivoting at row 2, col 1, the second Tableau is

x	y	u	v	w	P	const
0	$\frac{3}{5}$	1	$-\frac{1}{5}$	0	0	6
1	$\frac{2}{5}$	0	$\frac{1}{5}$	0	0	10
0	1	0	0	1	0	12
0	$-\frac{11}{5}$	0	$\frac{7}{5}$	0	1	70

Basic variables: x, u, w, P

Non-basic variables: y, v

Setting the non-basic variables to zero, the values of the basic variables are:

$x = 10$, $u = 6$, $w = 12$, $P = 70$

Since the only negative entry in the bottom row is $-\frac{11}{5}$ in column 2, the pivot column is 2.

16. Finding the Pivot Row

In the Simplex Tableau

x	y	u	v	w	P	const	ratio
0	$\frac{3}{5}$	1	$-\frac{1}{5}$	0	0	6	$\frac{6}{3/5} = 10$
1	$\frac{2}{5}$	0	$\frac{1}{5}$	0	0	10	$\frac{10}{2/5} = 25$
0	1	0	0	1	0	12	$\frac{12}{1} = 12$
0	$-\frac{11}{5}$	0	$\frac{7}{5}$	0	1	70	

The smallest ratio is 10 and occurs in row 1.

So the pivot row is 1.

17. Pivot at row 1, col 2

0	$\frac{3}{5}$	1	$-\frac{1}{5}$	0	0	6
1	$\frac{2}{5}$	0	$\frac{1}{5}$	0	0	10
0	1	0	0	1	0	12
0	$-\frac{11}{5}$	0	$\frac{7}{5}$	0	1	70

0	$\boxed{\frac{3}{5}}$	1	$-\frac{1}{5}$	0	0	6
1	$\frac{2}{5}$	0	$\frac{1}{5}$	0	0	10
0	1	0	0	1	0	12
0	$-\frac{11}{5}$	0	$\frac{7}{5}$	0	1	70

Pivot at row 1 col 2

0	$\frac{3}{5}$	1	$-\frac{1}{5}$	0	0	6
1	$\frac{2}{5}$	0	$\frac{1}{5}$	0	0	10
0	1	0	0	1	0	12
0	$-\frac{11}{5}$	0	$\frac{7}{5}$	0	1	70

Multiply row 1 by $\frac{5}{3}$

0	1	$\frac{5}{3}$	$-\frac{1}{3}$	0	0	10
1	$\frac{2}{5}$	0	$\frac{1}{5}$	0	0	10
0	1	0	0	1	0	12
0	$-\frac{11}{5}$	0	$\frac{7}{5}$	0	1	70

0	1	$\frac{5}{3}$	$-\frac{1}{3}$	0	0	10
1	$\frac{2}{5}$	0	$\frac{1}{5}$	0	0	10
0	1	0	0	1	0	12
0	$-\frac{11}{5}$	0	$\frac{7}{5}$	0	1	70

Add $-\frac{2}{5} \times$ row 1 to row 2

$$-\frac{2}{5} \times 1 + \frac{2}{5} = 0$$

$$-\frac{2}{5} \times \frac{5}{3} + 0 = -\frac{2}{3}$$

$$-\frac{2}{5} \times -\frac{1}{3} + \frac{1}{5} = \frac{1}{3}$$

$$-\frac{2}{5} \times 10 + 10 = 6$$

0	1	$\frac{5}{3}$	$-\frac{1}{3}$	0	0	10
1	0	$-\frac{2}{3}$	$\frac{1}{3}$	0	0	6
0	1	0	0	1	0	12
0	$-\frac{11}{5}$	0	$\frac{7}{5}$	0	1	70

0	1	$\frac{5}{3}$	$-\frac{1}{3}$	0	0	10
1	0	$-\frac{2}{3}$	$\frac{1}{3}$	0	0	6
0	1	0	0	1	0	12
0	$-\frac{11}{5}$	0	$\frac{7}{5}$	0	1	70

Add $-1 \times$ row 1 to row 3

$$-1 \times 1 + 1 = 0$$

$$-1 \times \frac{5}{3} + 0 = -\frac{5}{3}$$

$$-1 \times -\frac{1}{3} + 0 = \frac{1}{3}$$

$$-1 \times 10 + 12 = 2$$

0	1	$\frac{5}{3}$	$-\frac{1}{3}$	0	0	10
1	0	$-\frac{2}{3}$	$\frac{1}{3}$	0	0	6
0	0	$-\frac{5}{3}$	$\frac{1}{3}$	1	0	2
0	$-\frac{11}{5}$	0	$\frac{7}{5}$	0	1	70

0	1	$\frac{5}{3}$	$-\frac{1}{3}$	0	0	10
1	0	$-\frac{2}{3}$	$\frac{1}{3}$	0	0	6
0	0	$-\frac{5}{3}$	$\frac{1}{3}$	1	0	2
0	$-\frac{11}{5}$	0	$\frac{7}{5}$	0	1	70

Add $\frac{11}{5}$ $\times$ row 1 to row 4

$$\frac{11}{5} \times 1 + -\frac{11}{5} = 0$$

$$\frac{11}{5} \times \frac{5}{3} + 0 = \frac{11}{3}$$

$$\frac{11}{5} \times -\frac{1}{3} + \frac{7}{5} = \frac{2}{3}$$

$$\frac{11}{5} \times 10 + 70 = 92$$

0	1	$\frac{5}{3}$	$-\frac{1}{3}$	0	0	10
1	0	$-\frac{2}{3}$	$\frac{1}{3}$	0	0	6
0	0	$-\frac{5}{3}$	$\frac{1}{3}$	1	0	2
0	0	$\frac{11}{3}$	$\frac{2}{3}$	0	1	92

18. The Third Tableau

After pivoting at row 2, col 1, the third Tableau is

x	y	u	v	w	P	const
0	1	$\frac{5}{3}$	$-\frac{1}{3}$	0	0	10
1	0	$-\frac{2}{3}$	$\frac{1}{3}$	0	0	6
0	0	$-\frac{5}{3}$	$\frac{1}{3}$	1	0	2
0	0	$\frac{11}{3}$	$\frac{2}{3}$	0	1	92

Basic variables are: x, y, w, P

Non-basic variables are: u, v

Setting the non-basic variables to zero, it is easy to read the values of the basic variables:

$x = 6$, $y = 10$, $w = 2$, $P = 92$

Since there are no negative entries in the bottom row, we are done. The maximum value of P is 92 and occurs when $x = 6$ and $y = 10$.

19. How do we know we're done?

The bottom row of the Tableau is

x	y	u	v	w	P	const
0	0	$\frac{11}{3}$	$\frac{2}{3}$	0	1	92

This row corresponds to the equation:

$$0x + 0y + \frac{11}{3}u + \frac{2}{3}v + 0w + P = 92$$

or, equivalently, $P = 92 - \dfrac{11}{3}u - \dfrac{2}{3}v$

Since the slack variables u and v must be ≥ 0, assigning any positive value to either of them will just reduce the value of P.

Thus, the maximal value of P is 92.

20. Why does this work?

The preceding example starts with two non-basic variables: x and y. Setting the non-basic variables to zero gives us $x = 0$, $y = 0$, aka, the origin. For a standard linear programming problem the origin is always an element of the feasible set—though rarely is it the optimal solution. Whenever we pivot, we trade one non-basic variable for one basic variable.

In our example, our first pivot is in row 2, column 1, making x a basic variable and v a non-basic variable. The non-basic variables are now y and v. Setting y to zero gives us a point on the x-axis. Since $5x + 2y + v = 50$, setting v to zero gives us a point on the line $5x + 2y = 50$. By setting $y = 0$, $v = 0$ we obtain the x-interecept $(x, y) = (10, 0)$ of the line $5x + 2y = 50$.

21. Explanation Continued

We obtain the profit function from the bottom row of the second tableau

x	y	u	v	w	P	const
0	$-\frac{11}{5}$	0	$\frac{7}{5}$	0	1	70

Rewrite the equation $-\frac{11}{5}y + \frac{7}{5}v + P = 70$ as $P = 70 + \dfrac{11}{5}y - \dfrac{7}{5}v$

Increasing the value of y (which is currently 0) will increase the profit.

We pivot a second time at row 1, column 2.

Variable y now becomes a basic variable and switches with v which becomes non-basic.

The non-basic variables are u and v.

22. More Explanation

What does it mean when we set $u = 0$ and $v = 0$?

We have already seen that setting v to zero gives us a point on the line $5x + 2y = 50$.

Since $x+y+u = 16$, setting u to zero gives us a point on the line $x+y = 16$.

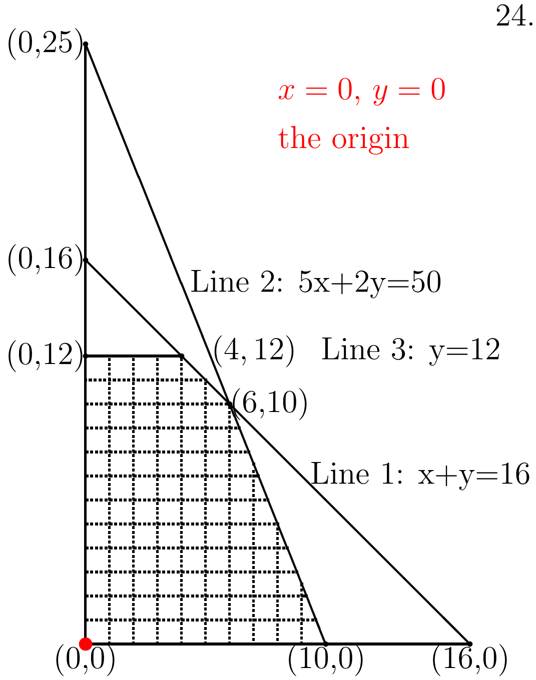
The (x, y) point for which $u = 0$, $v = 0$ is the intersection of these two lines, the point $(10, 6)$.

No further pivoting can increase the value of P—currently equal to 92—so we are done.

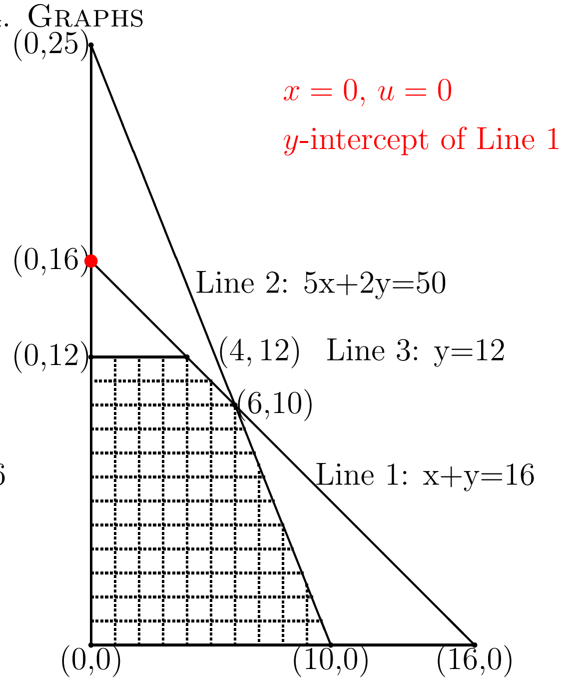
23. Important points

- we always start at a point in the feasible set $(x, y) = (0, 0)$.
- searching for the most negative entry in the bottom row indicates the variable to swap with one of the non-basic variables which "appears to" increase the profit by the largest factor.
- setting the non-basic variables to zero just gives us an intersection point of lines in the feasible set
- checking the ratio to find the pivot row prevents us from moving to a point which is outside the feasible region.

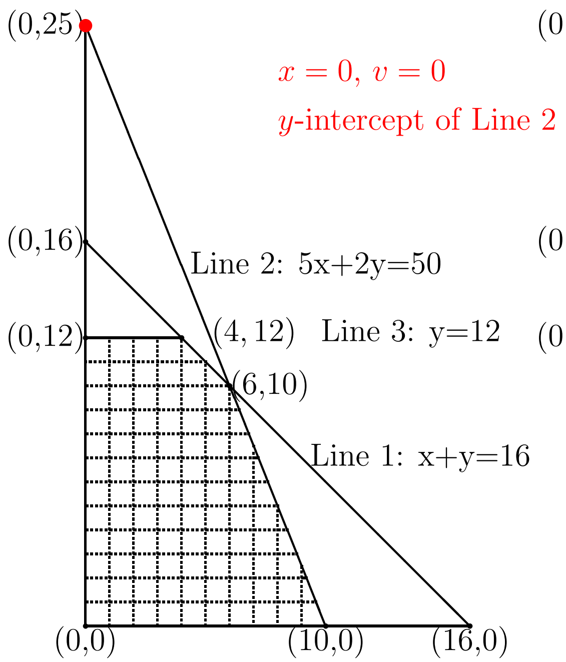
24. Graphs



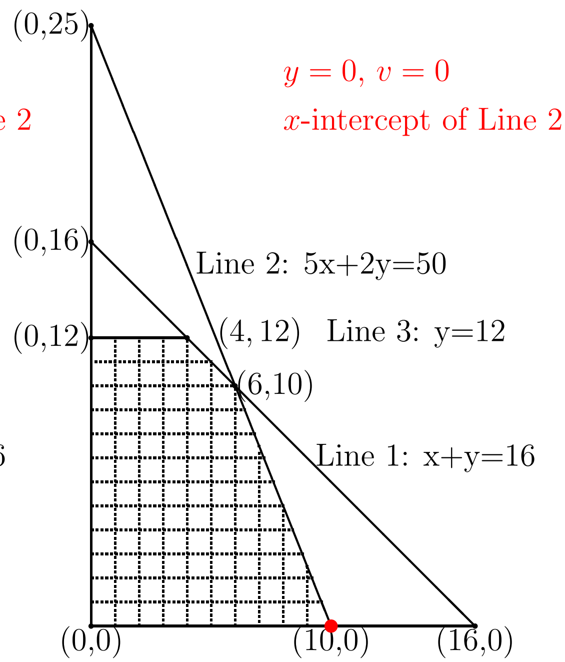
(0,25)
$x = 0,\ y = 0$
the origin
(0,16)
Line 2: 5x+2y=50
(0,12)
$(4, 12)$
Line 3: y=12
(6,10)
Line 1: x+y=16
(0,0)
(10,0)
(16,0)



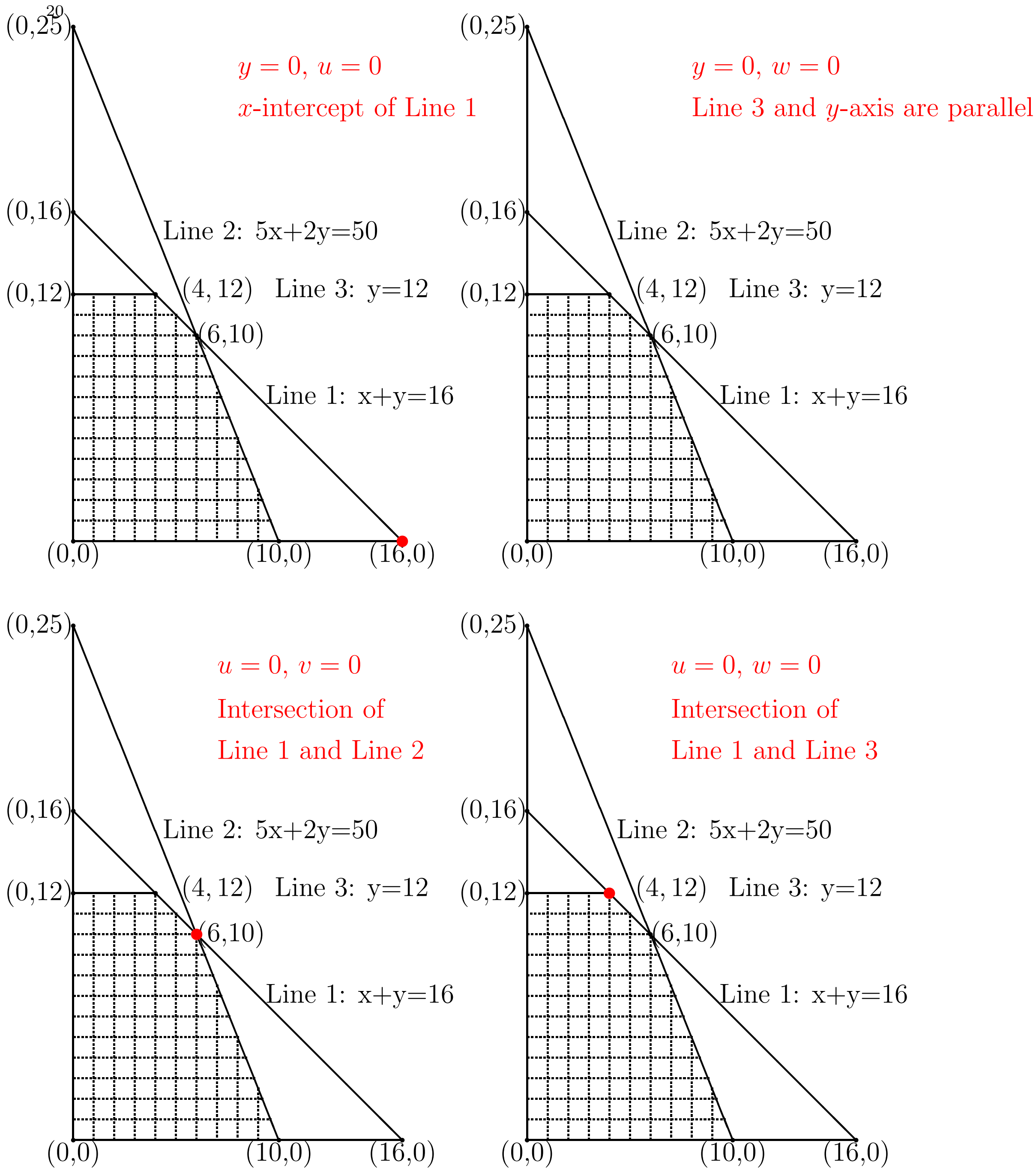
(0,25)
$x = 0,\ u = 0$
y-intercept of Line 1
(0,16)
Line 2: 5x+2y=50
(0,12)
$(4, 12)$
Line 3: y=12
(6,10)
Line 1: x+y=16
(0,0)
(10,0)
(16,0)

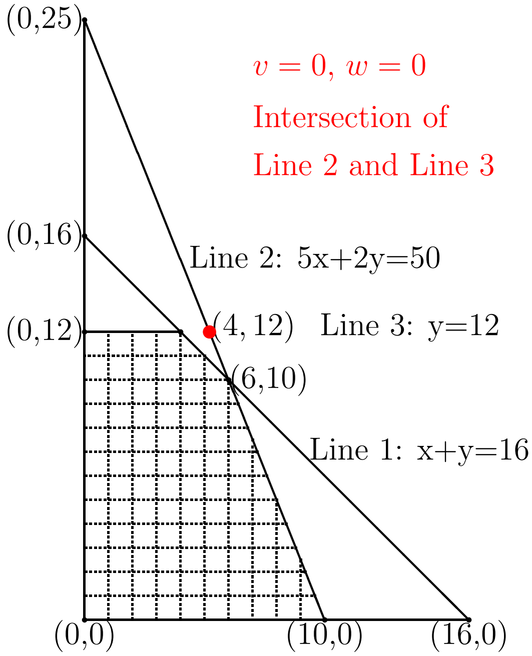


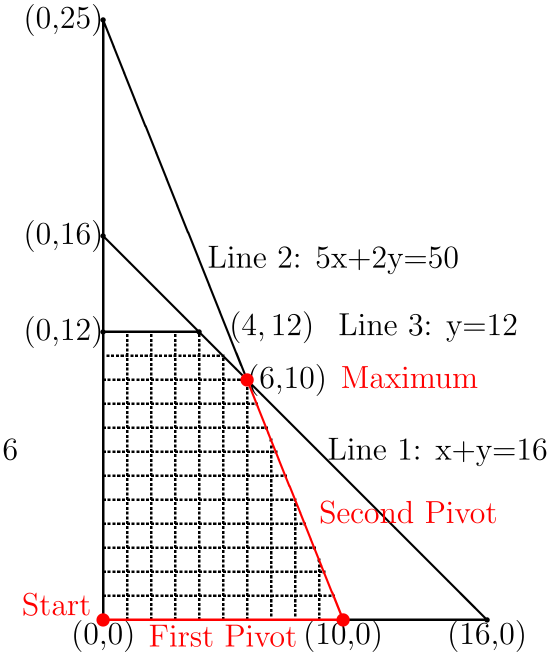
(0,25)
$x = 0,\ v = 0$
y-intercept of Line 2
(0,16)
Line 2: 5x+2y=50
(0,12)
$(4, 12)$
Line 3: y=12
(6,10)
Line 1: x+y=16
(0,0)
(10,0)
(16,0)



(0,25)
$y = 0,\ v = 0$
x-intercept of Line 2
(0,16)
Line 2: 5x+2y=50
(0,12)
$(4, 12)$
Line 3: y=12
(6,10)
Line 1: x+y=16
(0,0)
(10,0)
(16,0)


(0,25)
$y = 0,\ u = 0$
x-intercept of Line 1
(0,16)
Line 2: 5x+2y=50
(0,12)
(4, 12)
Line 3: y=12
(6,10)
Line 1: x+y=16
(0,0)
(10,0)
(16,0)
(0,25)
$y = 0,\ w = 0$
Line 3 and y-axis are parallel
(0,16)
Line 2: 5x+2y=50
(0,12)
(4, 12)
Line 3: y=12
(6,10)
Line 1: x+y=16
(0,0)
(10,0)
(16,0)
(0,25)
$u = 0,\ v = 0$
Intersection of
Line 1 and Line 2
(0,16)
Line 2: 5x+2y=50
(0,12)
(4, 12)
Line 3: y=12
(6,10)
Line 1: x+y=16
(0,0)
(10,0)
(16,0)
(0,25)
$u = 0,\ w = 0$
Intersection of
Line 1 and Line 3
(0,16)
Line 2: 5x+2y=50
(0,12)
(4, 12)
Line 3: y=12
(6,10)
Line 1: x+y=16
(0,0)
(10,0)
(16,0)





25. Construction Problem

Maximize

Profit: $P = 2400x + 3400y$

subject to the conditions:

Lot: $x + y \leq 150$

Capital $3x + 8y \leq 720$

Labor $3x + 4y \leq 480$

26. Introduce Slack Variables

Maximize

Profit: $P = 2400x + 3400y$

subject to the conditions:

Lot: $x + y + u = 150$

Capital $3x + 8y + v = 720$

Labor $3x + 4y + w = 480$

$x, y, u, v, w \geq 0$

27. Construction Problem Tableau

Rewriting the objective function $P = 2400x + 3400y$ as

$$-2400x + -3400y + P = 0$$

the Simplex Tableau becomes

x	y	u	v	w	P	const
1	1	1	0	0	0	150
3	8	0	1	0	0	720
3	4	0	0	1	0	480
−2400	−3400	0	0	0	1	0

The most negative entry in the bottom row is −3400 in column 2. So the pivot col is 2.

28. Find the First Pivot

In the Simplex Tableau

x	y	u	v	w	P	const	ratio
1	1	1	0	0	0	150	$\frac{150}{1} = 150$
3	8	0	1	0	0	720	$\frac{720}{8} = 90$
3	4	0	0	1	0	480	$\frac{480}{4} = 120$
−2400	−3400	0	0	0	1	0	

The smallest ratio is 90 and occurs in row 2.

So the pivot is at row 2, col 2.

29. Pivot at row 2, col 2

1	1	1	0	0	0	150
3	8	0	1	0	0	720
3	4	0	0	1	0	480
−2400	−3400	0	0	0	1	0

1	1	1	0	0	0	150
3	8	0	1	0	0	720
3	4	0	0	1	0	480
−2400	−3400	0	0	0	1	0

Pivot at row 2 col 2

1	1	1	0	0	0	150
3	8	0	1	0	0	720
3	4	0	0	1	0	480
−2400	−3400	0	0	0	1	0

Multiply row 2 by $\frac{1}{8}$

1	1	1	0	0	0	150
$\frac{3}{8}$	1	0	$\frac{1}{8}$	0	0	90
3	4	0	0	1	0	480
−2400	−3400	0	0	0	1	0

1	1	1	0	0	0	150
$\frac{3}{8}$	1	0	$\frac{1}{8}$	0	0	90
3	4	0	0	1	0	480
−2400	−3400	0	0	0	1	0

Add $-1 \times$ row 2 to row 1

$$-1 \times \frac{3}{8} + 1 = \frac{5}{8}$$

$$-1 \times 1 + 1 = 0$$

$$-1 \times \frac{1}{8} + 0 = -\frac{1}{8}$$

$$-1 \times 90 + 150 = 60$$

$\frac{5}{8}$	0	1	$-\frac{1}{8}$	0	0	60
$\frac{3}{8}$	1	0	$\frac{1}{8}$	0	0	90
3	4	0	0	1	0	480
-2400	-3400	0	0	0	1	0

$\frac{5}{8}$	0	1	$-\frac{1}{8}$	0	0	60
$\frac{3}{8}$	1	0	$\frac{1}{8}$	0	0	90
3	4	0	0	1	0	480
-2400	-3400	0	0	0	1	0

Add $-4 \times$ row 2 to row 3

$$-4 \times \frac{3}{8} + 3 = \frac{3}{2}$$

$$-4 \times 1 + 4 = 0$$

$$-4 \times \frac{1}{8} + 0 = -\frac{1}{2}$$

$$-4 \times 90 + 480 = 120$$

$\frac{5}{8}$	0	1	$-\frac{1}{8}$	0	0	60
$\frac{3}{8}$	1	0	$\frac{1}{8}$	0	0	90
$\frac{3}{2}$	0	0	$-\frac{1}{2}$	1	0	120
-2400	-3400	0	0	0	1	0

$\frac{5}{8}$	0	1	$-\frac{1}{8}$	0	0	60
$\frac{3}{8}$	1	0	$\frac{1}{8}$	0	0	90
$\frac{3}{2}$	0	0	$-\frac{1}{2}$	1	0	120
-2400	-3400	0	0	0	1	0

Add 3400 × row 2 to row 4

$$3400 \times \frac{3}{8} + -2400 = -1125$$

$$3400 \times 1 + -3400 = 0$$

$$3400 \times \frac{1}{8} + 0 = 425$$

$$3400 \times 90 + 0 = 306000$$

$\frac{5}{8}$	0	1	$-\frac{1}{8}$	0	0	60
$\frac{3}{8}$	1	0	$\frac{1}{8}$	0	0	90
$\frac{3}{2}$	0	0	$-\frac{1}{2}$	1	0	120
-1125	0	0	425	0	1	306000

30. Find the Second Pivot

The only negative entry in the bottom row is -1125 in col 1; so the pivot column is 1.

x	y	u	v	w	P	const	ratio
$\frac{5}{8}$	0	1	$-\frac{1}{8}$	0	0	60	$\frac{60}{5/8} = 96$
$\frac{3}{8}$	1	0	$\frac{1}{8}$	0	0	90	$\frac{90}{3/8} = 240$
$\frac{3}{2}$	0	0	$-\frac{1}{2}$	1	0	120	$\frac{120}{3/2} = 80$
-1125	0	0	425	0	1	$306K$	

The smallest ratio is 80 and occurs in row 3.

So the pivot is at row 3, col 1.

31. Pivot at row 3, col 1

$\frac{5}{8}$	0	1	$-\frac{1}{8}$	0	0	60
$\frac{3}{8}$	1	0	$\frac{1}{8}$	0	0	90
$\frac{3}{2}$	0	0	$-\frac{1}{2}$	1	0	120
-1125	0	0	425	0	1	306000

$\frac{5}{8}$	0	1	$-\frac{1}{8}$	0	0	60
$\frac{3}{8}$	1	0	$\frac{1}{8}$	0	0	90
$\boxed{\frac{3}{2}}$	0	0	$-\frac{1}{2}$	1	0	120
-1125	0	0	425	0	1	306000

Pivot at row 3 col 1

$\frac{5}{8}$	0	1	$-\frac{1}{8}$	0	0	60
$\frac{3}{8}$	1	0	$\frac{1}{8}$	0	0	90
$\frac{3}{2}$	0	0	$-\frac{1}{2}$	1	0	120
-1125	0	0	425	0	1	306000

Multiply row 3 by $\frac{2}{3}$

$\frac{5}{8}$	0	1	$-\frac{1}{8}$	0	0	60
$\frac{3}{8}$	1	0	$\frac{1}{8}$	0	0	90
1	0	0	$-\frac{1}{3}$	$\frac{2}{3}$	0	80
−1125	0	0	425	0	1	306000

$\frac{5}{8}$	0	1	$-\frac{1}{8}$	0	0	60
$\frac{3}{8}$	1	0	$\frac{1}{8}$	0	0	90
1	0	0	$-\frac{1}{3}$	$\frac{2}{3}$	0	80
−1125	0	0	425	0	1	306000

Add $-\frac{5}{8} \times$ row 3 to row 1

$$-\frac{5}{8} \times 1 + \frac{5}{8} = 0$$

$$-\frac{5}{8} \times -\frac{1}{3} + -\frac{1}{8} = \frac{1}{12}$$

$$-\frac{5}{8} \times \frac{2}{3} + 0 = -\frac{5}{12}$$

$$-\frac{5}{8} \times 80 + 60 = 10$$

0	0	1	$\frac{1}{12}$	$-\frac{5}{12}$	0	10
$\frac{3}{8}$	1	0	$\frac{1}{8}$	0	0	90
1	0	0	$-\frac{1}{3}$	$\frac{2}{3}$	0	80
-1125	0	0	425	0	1	306000

0	0	1	$\frac{1}{12}$	$-\frac{5}{12}$	0	10
$\frac{3}{8}$	1	0	$\frac{1}{8}$	0	0	90
1	0	0	$-\frac{1}{3}$	$\frac{2}{3}$	0	80
-1125	0	0	425	0	1	306000

Add $-\frac{3}{8} \times$ row 3 to row 2

$$-\frac{3}{8} \times 1 + \frac{3}{8} = 0$$

$$-\frac{3}{8} \times -\frac{1}{3} + \frac{1}{8} = \frac{1}{4}$$

$$-\frac{3}{8} \times \frac{2}{3} + 0 = -\frac{1}{4}$$

$$-\frac{3}{8} \times 80 + 90 = 60$$

0	0	1	$\frac{1}{12}$	$-\frac{5}{12}$	0	10
0	1	0	$\frac{1}{4}$	$-\frac{1}{4}$	0	60
1	0	0	$-\frac{1}{3}$	$\frac{2}{3}$	0	80
-1125	0	0	425	0	1	306000

0	0	1	$\frac{1}{12}$	$-\frac{5}{12}$	0	10
0	1	0	$\frac{1}{4}$	$-\frac{1}{4}$	0	60
1	0	0	$-\frac{1}{3}$	$\frac{2}{3}$	0	80
-1125	0	0	425	0	1	306000

Add 1125 × row 3 to row 4

$$1125 \times 1 + -1125 = 0$$

$$1125 \times -\frac{1}{3} + 425 = 50$$

$$1125 \times \frac{2}{3} + 0 = 750$$

$$1125 \times 80 + 306000 = 396000$$

0	0	1	$\frac{1}{12}$	$-\frac{5}{12}$	0	10
0	1	0	$\frac{1}{4}$	$-\frac{1}{4}$	0	60
1	0	0	$-\frac{1}{3}$	$\frac{2}{3}$	0	80
0	0	0	50	750	1	396000

32. The Final Tableau

x	y	u	v	w	P	const
0	0	1	$\frac{1}{12}$	$-\frac{5}{12}$	0	10
0	1	0	$\frac{1}{4}$	$-\frac{1}{4}$	0	60
1	0	0	$-\frac{1}{3}$	$\frac{2}{3}$	0	80
0	0	0	50	750	1	$396K$

Basic variables: x, y, u, P

Non-basic variables: v, w

Setting the non-basic variables to zero, the values of the basic variables are:

$x = 80$, $y = 60$, $u = 10$, $P = 396,000$

Since there are no negative entries in the bottom row, we are done. The maximum value of P is 396,000 and occurs when $x = 80$ and $y = 60$.