1. THE TRANSPORTATION PROBLEM
Minimize the cost \[ C = -3x - 2y + 375 \]
subject to the constraints
\[ \bullet \ x \leq 25, \ y \leq 30 \]
\[ \bullet \ x + y \geq 15 \]
\[ \bullet \ x + y \leq 45 \]
\[ \bullet \ x \geq 0, \ y \geq 0 \]
This is an example of a non-standard problem. Some inequalities are \( \leq \) and some are \( \geq \).

2. TWO TRICKS
To convert a problem in non-standard form into standard form we utilize two tricks:
\[ \bullet \ \text{Minimizing} \ F \ \text{is equivalent to maximizing} \ -F \]
\[ \bullet \ \text{and vice-versa} \]
\[ \bullet \ \text{You can change an inequality from} \ \geq \ \text{to} \ \leq \ \text{by multiplying both sides by} \ -1 \]
\[ \bullet \ \text{A similarly trick changes from} \ \leq \ \text{to} \ \geq \]

3. APPLYING THE TWO TRICKS
To convert the transportation problem into standard form we utilize these tricks:
Instead of minimizing \( C = -3x - 2y + 375 \),
maximize the negative \( F = -C = 3x + 2y - 375 \)
All but one constraining inequality has the correct direction for maximization (\( \leq \)): \[ x + y \geq 15 \]
Multiplying this inequality by \(-1\) gives
\[-x - y \leq -15 \]

4. REVISED TRANSPORTATION PROBLEM
Maximize the function \[ F = 3x + 2y - 375 \]
subject to the constraints
\[ \bullet \ x \leq 25, \ y \leq 30 \]
\[ \bullet \ x + y \leq 45 \]
\[ \bullet \ -x + -y \leq -15 \]
\[ \bullet \ x \geq 0, \ y \geq 0 \]
Using slack variables \( u, v, w, \) and \( t \):
\[ \bullet \ x + u = 25, \ y + v = 30 \]
\[ \bullet \ x + y + w = 45 \]
\[ \bullet \ -x + -y + t = -15 \]
\[ \bullet \ x \geq 0, \ y \geq 0 \]
5. Simplex Tableau

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<th>y</th>
<th>u</th>
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Setting the non-basic variables \((x\) and \(y)\) to zero gives the following values for the basic variables:
\[u = 25 \quad v = 30 \quad w = 45 \quad t = -15\]
OOPS! One of our slack variables is negative. This means that the initial point \((x, y) = (0, 0)\) is not in the feasible set.

6. Fixing a non-Standard Tableau

When an entry in the constant row (not at the bottom) is negative, we can fix it by the following procedure:

- Select one of the negative entries in its row. The column containing this entry will be the pivot column.
- Select the pivot row by computing the **positive** ratio of elements in the constant column and elements in the pivot column (except at the bottom). The smallest positive ratio determines the pivot row.
- Pivot
- Repeat until no negative entries are found in the constant column of the resulting tableau.

7. Finding Pivot Column

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</tr>
</tbody>
</table>

Since \(-15\) is the only negative entry in the constant column (**We do not count \(-375\) at the bottom**) we scan this row looking for another negative entry.

We find two: in columns 1 and column 2.

Flipping a coin, we choose column 1 to be the pivot column.

8. Finding the Pivot Row

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Since the smallest positive ratio 15 occurs in row 4, the pivot row is 4.

Pivot at row 4, col 1.
### 9. Pivot at row 4, col 1

#### Step 1. The Initial Tableau.

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#### Step 2.

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#### Pivot at row 4 col 1

#### Step 3.

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#### Step 4.

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Multiply row 4 by $-1$
### Step 5.

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Add $-1 \times$ row 4 to row 1

$-1 \times 1 + 1 = 0$

$-1 \times 1 + 0 = -1$

$-1 \times -1 + 0 = 1$

$-1 \times 15 + 25 = 10$

### Step 6.

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Add $-1 \times$ row 4 to row 3

$-1 \times 1 + 1 = 0$

$-1 \times 1 + 1 = 0$

$-1 \times -1 + 0 = 1$

$-1 \times 15 + 45 = 30$

### Step 8.

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Step 9.

\[
\begin{array}{ccccccc}
0 & -1 & 1 & 0 & 0 & 1 & 0 & 10 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 30 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 30 \\
1 & 1 & 0 & 0 & 0 & -1 & 0 & 15 \\
-3 & -2 & 0 & 0 & 0 & 0 & 1 & -375 \\
\end{array}
\]

Add \(3 \times \) row 4 to row 5

\[
\begin{align*}
3 \times 1 + -3 &= 0 \\
3 \times 1 + -2 &= 1 \\
3 \times -1 + 0 &= -3 \\
3 \times 15 + -375 &= -330
\end{align*}
\]

Step 10.

\[
\begin{array}{ccccccc}
0 & -1 & 1 & 0 & 0 & 1 & 0 & 10 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 30 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 30 \\
1 & 1 & 0 & 0 & 0 & -1 & 0 & 15 \\
0 & 1 & 0 & 0 & 0 & -3 & 1 & -330 \\
\end{array}
\]

10. Second Simplex Tableau

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</tr>
</tbody>
</table>

Since the only negative entry in the bottom row (we don’t count \(-330\)) is the \(-3\) in column 6, the pivot column is 6.

11. Finding Pivot Row

<table>
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<tr>
<th></th>
<th>(x)</th>
<th>(y)</th>
<th>(u)</th>
<th>(v)</th>
<th>(w)</th>
<th>(t)</th>
<th>(F)</th>
<th>const</th>
<th>ratio</th>
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<td>10</td>
<td>(\text{nn} ) (= 10)</td>
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<td>(\text{nn} ) (= 30)</td>
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</tr>
</tbody>
</table>

Since the minimum ratio of 10 occurs in row 1, the pivot row is 1.
Pivot at row 1, column 6.
12. **Pivot at row 1, col 6**

### Step 1. The Initial Tableau.

<table>
<thead>
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### Step 2.

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### Step 3.

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</table>

**Add 0 \times row 1 to row 2**

\[0 \times -1 + 1 = 1\]

\[0 \times 1 + 0 = 0\]

\[0 \times 1 + 0 = 0\]

\[0 \times 10 + 30 = 30\]

### Step 4.

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<td>-3</td>
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<td>-330</td>
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</tbody>
</table>
Step 5.

Add $-1 \times$ row 1 to row 3
$-1 \times -1 + 0 = 1$
$-1 \times 1 + 0 = -1$
$-1 \times 1 + 1 = 0$
$-1 \times 10 + 30 = 20$

Step 7.

Add $1 \times$ row 1 to row 4
$1 \times -1 + 1 = 0$
$1 \times 1 + 0 = 1$
$1 \times 1 + -1 = 0$
$1 \times 10 + 15 = 25$
Step 9.

Add $3 \times$ row 1 to row 5

$3 \times -1 + 1 = -2$

$3 \times 1 + 0 = 3$

$3 \times 1 + -3 = 0$

$3 \times 10 + -330 = -300$

13. **Third Simplex Tableau**

<table>
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<tr>
<th>$x$</th>
<th>$y$</th>
<th>$u$</th>
<th>$v$</th>
<th>$w$</th>
<th>$t$</th>
<th>$F$</th>
<th>const</th>
</tr>
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<tbody>
<tr>
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</table>

Since the minimum ratio is 20 occurs in row 3, the pivot row is 3.

Pivot at row 3, column 2.

14. **Find pivot row**

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<th>$u$</th>
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<td>$\frac{30}{30} = 30$</td>
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<td>$\frac{20}{10} = 20$</td>
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</table>

Setting the non-basic variables $y$ and $u$ to zero, the values of the basic variables are:

$x = 25$, $v = 30$, $w = 20$, $t = 10$, $F = -300$
15. **Pivot at row 3, col 2**

**Step 1. The Initial Tableau.**

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**Step 2.**

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**Step 3.**

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**Step 4.**

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</table>

Add $1 \times$ row 3 to row 1

$1 \times 1 + -1 = 0$

$1 \times -1 + 1 = 0$

$1 \times 1 + 0 = 1$

$1 \times 20 + 10 = 30$
### Step 5.

<p>| | | | | | | | | |</p>
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</table>

Add $-1 \times$ row 3 to row 2

$-1 \times 1 + 1 = 0$

$-1 \times -1 + 0 = 1$

$-1 \times 1 + 0 = -1$

$-1 \times 20 + 30 = 10$

### Step 7.

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### Step 6.

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### Step 8.

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<td>30</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>1</td>
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<td>1</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>-260</td>
<td></td>
</tr>
</tbody>
</table>

Add $2 \times$ row 3 to row 5

$2 \times 1 + -2 = 0$

$2 \times -1 + 3 = 1$

$2 \times 1 + 0 = 2$

$2 \times 20 + -300 = -260$
16. Final Simplex Tableau

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>u</th>
<th>v</th>
<th>w</th>
<th>t</th>
<th>F</th>
<th>const</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>10</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>20</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>25</td>
<td>20</td>
<td></td>
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<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>-260</td>
<td></td>
</tr>
</tbody>
</table>

Setting the non-basic variables $u$ and $w$ to zero, the values of the basic variables are: $x = 25$, $y = 20$, $v = 10$, $t = 30$, $F = -260$

Since there are no negative entries in the bottom row (before the constant column), we are done.

Since the cost function $C$ of the original transportation problem is the negative of $F$, Minimum cost is $260$ at the point $(x, y) = (25, 20)$

17. Three Types of Problems

(1) Standard Form Maximization
   - Objective Function is maximized
   - All inequalities are $\leq$ (except $x, y \geq 0$)
   - Use Standard Simplex Method
   - Prototype: Construction Problem

(2) Standard Form Minimization
   - Objective Function is minimized
   - All inequalities are $\geq$
   - Reverse rows and columns of the data table to obtain the dual Problem
   - Use Simplex Method on Dual Problem
   - Prototype: Nutrition Problem

(3) Non-Standard Form
   - Inequalities are mixed (some $\leq$, some $\geq$)
   - Convert the problem to standard form
   - Pivot to remove negative entries in the constant column
   - Prototype: Transportation Problem

18. Standard Form Problem

Construction Problem
Maximize profit $P = 2400x + 3400y$ subject to
$x + y \leq 150$
$3x + 8y \leq 720$
$3x + 4y \leq 480$
$x, y \geq 0$

This problem is in standard form: it is a maximization problem and all inequalities are $\leq$.

19. Construction Tableau #1

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>u</th>
<th>v</th>
<th>w</th>
<th>P</th>
<th>const</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>720</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>480</td>
<td></td>
</tr>
<tr>
<td>-2400</td>
<td>-3400</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

$x = 0$  $y = 0$  $P = 0$

Pivot at row 2, col 2.

20. Construction Tableau #2

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>u</th>
<th>v</th>
<th>w</th>
<th>P</th>
<th>const</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>0</td>
<td>-5</td>
<td>1</td>
<td>0</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td>-1125</td>
<td>0</td>
<td>0</td>
<td>425</td>
<td>0</td>
<td>1</td>
<td>306K</td>
<td></td>
</tr>
</tbody>
</table>

$x = 0$  $y = 90$  $P = 306K$

Pivot at row 3, col 1.

21. Construction Tableau #3

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>u</th>
<th>v</th>
<th>w</th>
<th>P</th>
<th>const</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>12</td>
<td>-5</td>
<td>0</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-12</td>
<td>0</td>
<td>0</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-5</td>
<td>3</td>
<td>0</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>50</td>
<td>750</td>
<td>1</td>
<td>396K</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$x = 80$  $y = 60$

$P = 396,000$ (maximum)
22. Dual Problem

The Nutrition Problem
Minimize $C = 21x + 14y$ subject to
\[2x + 3y \geq 12\]
\[3x + y \geq 6\]
\[x + 3y \geq 9\]
\[x, y \geq 0\]

Dual:
Maximize $F = 12u + 6v + 9w$ subject to
\[2u + 3v + w \leq 21\]
\[3u + v + 3w \leq 14\]
\[u, v, w \geq 0\]

23. Nutrition Tableau #1

<table>
<thead>
<tr>
<th></th>
<th>u</th>
<th>v</th>
<th>w</th>
<th>x</th>
<th>y</th>
<th>F</th>
<th>const</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
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<td>1</td>
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<td>0</td>
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<td>21</td>
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<tr>
<td></td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>-12</td>
<td>-6</td>
<td>-9</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Value of the variables for the original minimization are read from the bottom of the tableau:
\[x = 0\quad y = 0\]

24. Nutrition Tableau #2

<table>
<thead>
<tr>
<th></th>
<th>u</th>
<th>v</th>
<th>w</th>
<th>x</th>
<th>y</th>
<th>F</th>
<th>const</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>$\frac{7}{3}$</td>
<td>-1</td>
<td>1</td>
<td>$-\frac{2}{3}$</td>
<td>0</td>
<td>$\frac{35}{3}$</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>$\frac{1}{3}$</td>
<td>1</td>
<td>0</td>
<td>$\frac{1}{3}$</td>
<td>0</td>
<td>$\frac{14}{3}$</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>-2</td>
<td>3</td>
<td>10</td>
<td>4</td>
<td>1</td>
<td>56</td>
</tr>
</tbody>
</table>

\[x = 10\quad y = 4\]

25. Nutrition Tableau #3

<table>
<thead>
<tr>
<th></th>
<th>u</th>
<th>v</th>
<th>w</th>
<th>x</th>
<th>y</th>
<th>F</th>
<th>const</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>$-\frac{3}{7}$</td>
<td>$\frac{2}{7}$</td>
<td>-1</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0</td>
<td>$\frac{3}{7}$</td>
<td>$-\frac{1}{7}$</td>
<td>$\frac{3}{7}$</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>$\frac{12}{7}$</td>
<td>$\frac{9}{7}$</td>
<td>$\frac{23}{7}$</td>
<td>1</td>
<td>66</td>
</tr>
</tbody>
</table>

\[x = \frac{6}{7}\quad y = \frac{2}{7}\]

By principal of duality, minimum $C =$ maximum $F = 66$

26. Non Standard Problem

Original Transportation Problem:
Minimize the cost $C = -3x - 2y + 375$ subject to
\[x \leq 25, y \leq 30\]
\[x + y \geq 15\]
\[x + y \leq 45\]
\[x \geq 0, y \geq 0\]

Non-standard because some inequalities are $\leq$ and some are $\geq$.

Revised Transportation Problem:
Maximize the function $F = 3x + 2y - 375$ subject to
\[x \leq 25, y \leq 30\]
\[-x - y \leq -15\]
\[x + y \leq 45\]
\[x \geq 0, y \geq 0\]

27. Transportation Tableau #1

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>u</th>
<th>v</th>
<th>w</th>
<th>t</th>
<th>F</th>
<th>const</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0</td>
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<td>25</td>
</tr>
<tr>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>45</td>
</tr>
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<td>1</td>
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</tr>
<tr>
<td></td>
<td>-3</td>
<td>-2</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-375</td>
</tr>
</tbody>
</table>

\[x = 0\quad y = 0\quad F = -375\]

Conflict: $-15$ in constant column
Resolve conflict by pivoting at row 4, col 1.
28. **Transportation Tableau #2**

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>u</th>
<th>v</th>
<th>w</th>
<th>t</th>
<th>F</th>
<th>const</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>10</td>
<td></td>
</tr>
<tr>
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<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
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<td></td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>-3</td>
<td>1</td>
<td>-330</td>
<td></td>
</tr>
</tbody>
</table>

\( x = 15 \quad y = 0 \quad F = -330 \)

no negatives in constant column

(above bottom row)

Pivot at row 1, col 6.

---

29. **Transportation Tableau #3**

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>u</th>
<th>v</th>
<th>w</th>
<th>t</th>
<th>F</th>
<th>const</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>10</td>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</tr>
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<td>-1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>20</td>
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<td>25</td>
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</tr>
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<td>3</td>
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<td>0</td>
<td>0</td>
<td>1</td>
<td>-300</td>
<td></td>
</tr>
</tbody>
</table>

\( x = 25 \quad y = 0 \quad F = -300 \)

Pivot at row 3, col 2.

---

30. **Transportation Tableau #4**

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>u</th>
<th>v</th>
<th>t</th>
<th>F</th>
<th>const</th>
</tr>
</thead>
<tbody>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>0</td>
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<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
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<td>1</td>
<td>0</td>
<td>0</td>
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<td>25</td>
</tr>
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<td>0</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>-260</td>
</tr>
</tbody>
</table>

\( x = 25 \quad y = 20 \quad F = -260 \)

Solution: Since \( C = -F \),
Minimum cost is $260