

MATH 210 LECTURE NOTES:  
TEN PROBABILITY  
REVIEW PROBLEMS

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1. REVIEW QUESTION 1

“Face card” means “jack, queen, or king.”

You draw two cards from a standard 52-card deck.

Find the probability that

- the first card is not a face card
- the second card is not a face card given that the first card is not a face card
- both cards are not face cards
- at least one of the cards is a face card.

2. QUESTION 1 SOLUTION

Find the probability that

- the first card is not a face card  
 $1 - \frac{12}{52} = 1 - \frac{3}{13} = \frac{10}{13}$
- the second card is not a face card given that the first card is not a face card  
 $\frac{39}{51} = \frac{3 \cdot 13}{3 \cdot 17} = \frac{13}{17}$

3. QUESTION 1 SOLUTION CONT

Find the probability that

- both cards are not face cards  
 $P(\text{both are not face cards}) = P(\text{first card not face}) \cdot P(\text{second card not face} \mid \text{first card not face}) = \frac{10}{13} \cdot \frac{13}{17} = \frac{10}{17}$
- at least one of the cards is a face card.  $P(\text{at least one face card}) = 1 - P(\text{both are not face cards}) = 1 - \frac{10}{17} = \frac{7}{17}$

#### 4. REVIEW QUESTION 2

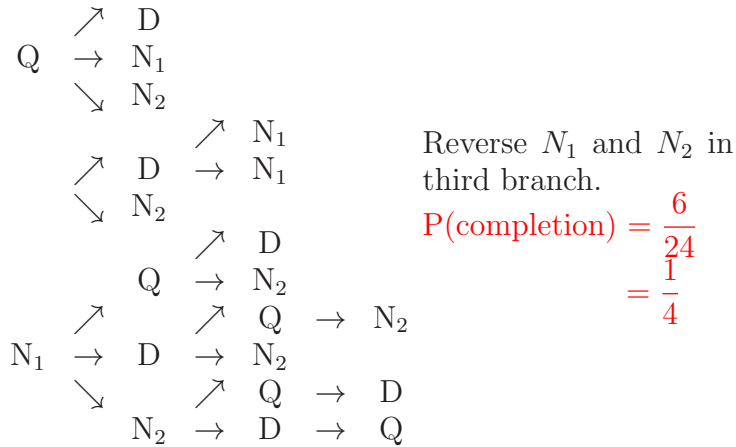
A quarter, a dime, and two nickels are placed in a box.

Coins are drawn out one at a time.

The drawing continues until a coin is drawn which is of smaller value than the one just previously drawn, or until all the coins are drawn.

What is the probability that all four coins will be drawn?

#### 5. QUESTION 2 SOLUTION



#### 6. REVIEW QUESTION 3

An urn contains seven red and three green balls.

A second urn contains five red and five green balls.

A ball is selected at random from the first urn and placed in the second.

Then a ball is selected at random from the second urn.

What is the probability of drawing a green ball the first time and a red ball the second time?

### 7. QUESTION 3 SOLUTION

Urn 1: seven red and three green balls.

Urn 2: five red and five green balls.

A ball is selected at random from Urn 1 and placed in Urn 2. Then a ball is selected from Urn 2.

P(first ball green and second ball is red)

= P(first ball green) · P(second ball is red | first ball green)

$$= \frac{3}{10} \cdot \frac{5}{11} = \frac{3}{22}$$

### 8. REVIEW QUESTION 4

The probability of any particular used marker being good (that is, it actually writes clearly on the board) is 15%.

Seven used markers are in a tray by the whiteboard.

What is the probability that exactly two of these seven markers are good?

### 9. QUESTION 4 SOLUTION

By the Binomial Distribution Theorem,

P(exactly 2 out of 7 is good)

$$= C(7, 2)(0.15)^2(0.85)^5$$

$$= .20965$$

## 10. REVIEW QUESTION 5

A bag contains 5 green and 8 red balls.

A man is condemned to draw a ball and to be executed if it is red one.

The sentence is subsequently mitigated in that

the condemned man is now allowed to divide the balls between two bags according to his own preference.

He is then blindfolded and made to choose one of the bags and then draw a ball from it.

What is, from his point of view, the most favorable division of the balls?

## 11. QUESTION 5 SOLUTION

A bag contains 5 green and 8 red balls.

The condemned man should divide the balls into two bags as follows:

Bag 1: 1 green ball

Bag 2: 4 green, 8 red balls

## 12. REVIEW QUESTION 6

Events A and B occur with the probabilities:

$$P(A) = \frac{17}{40} \quad P(B) = \frac{13}{59} \quad P(A \cap B) = \frac{2}{11}$$

Find

- $P(A \cup B)$
- $P(A \cap B^c)$
- $P(A | B)$
- Are A and B independent events?

## 13. QUESTION 6 SOLUTION

$$P(A) = \frac{17}{40} \quad P(B) = \frac{13}{59} \quad P(A \cap B) = \frac{2}{11}$$

- $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{17}{40} + \frac{13}{59} - \frac{2}{11}$
- $P(A \cap B^c) = P(A) - P(A \cap B) = \frac{17}{40} - \frac{2}{11}$
- $P(A | B) = \frac{P(A \cap B)}{P(B)} = \frac{2/11}{13/59} = \frac{2}{11} \cdot \frac{59}{13} = \frac{118}{143}$
- Are A and B independent events? No  $P(A | B) = \frac{118}{143} \neq P(A) = \frac{17}{40}$

## 14. REVIEW QUESTION 7

In a survey, 60 Catholics and 40 Protestants were asked whether they believe in abortion.

Five-twelvths of the Catholics said “yes,”

while 15 of the Protestants said “yes.”

- What is the probability that a person picked at random from the survey said “yes?”
- If you already know the person responded “yes,” what is the probability that he or she is a Catholic?

## 15. REVIEW QUESTION 7

60 Catholics and 40 Protestants surveyed.

100 = 60 + 40 people in the survey.

Five-twelvths of the Catholics said “yes”

$$\frac{5}{12}60 = 25$$

$$P(\text{a person picked at random from the survey said “yes?”}) = \frac{25 + 15}{100} = \frac{40}{100}$$

$$P(\text{person is Catholic given that the person responded "yes"}) = \frac{25}{40} = \frac{5}{8}$$

#### 16. REVIEW QUESTION 8

Among the numbers 1, 2, 3, 4, we first choose one at random;  
among the remaining numbers we then choose another.

Calculate the probability of picking an odd number

- at the first draw
- at the second draw
- at both draws.

#### 17. QUESTION 8 SOLUTION

First number chosen from 1, 2, 3, 4

Second number chosen from the remaining 3 numbers.

$$P(\text{first number is odd}) = \frac{2}{4} = \frac{1}{2}$$

$$P(\text{second number is odd}) = \frac{2}{4} = \frac{1}{2}$$

**Time Out!** Maybe you are thinking that the selection of the second ball should be from 3 balls, not 4. So why is the denominator 4?

#### 18. QUESTION 8 SOLUTION CONT'D

In asking “What is the probability the second ball is odd?” no conditions are given on the first ball, This question does not ask “What is the probability the second ball is odd given that the first ball is odd.” Without conditions there is no reason that any one of the four numbers 1—4 should be more likely to occur for the second ball than the others.

Still not convinced? Write out the all twelve ways that the two balls could be picked:

(1,2) (1,3) (1,4) (2,1) (2,3) (2,4)  
 (3,1) (3,2) (3,4) (4,1) (4,2) (4,3)

In 6 of these 12 cases, the second ball is odd.

- $P(\text{both odd}) = P(\text{first odd}) \times P(\text{second odd} \mid \text{first odd}) = \frac{2}{4} \times \frac{1}{3} = \frac{1}{6}$

### 19. REVIEW QUESTION 9

A number is chosen at random from the first 10,000 positive integers.

Find the probability that the number is:

- a perfect square (1, 4, 9, 16, 25, etc)
- divisible by 2 but not by 10
- a three digit number ending in 7 or 9
- divisible by 100 if you already know it is divisible by 250.

### 20. QUESTION 9 SOLUTION

A number is chosen at random from the first 10,000 positive integers.

- $P(\text{perfect square}) = \frac{100}{10,000} = \frac{1}{100}$
- $P(\text{divisible by 2 but not by 10}) = \frac{1}{2} - \frac{1}{10} = \frac{4}{10}$
- $P(\text{a three digit number ending in 7 or 9}) = \frac{9 \cdot 10 \cdot 2}{10,000} = \frac{9}{500}$
- $P(\text{divisible by 100 if you already know it is divisible by 250}) = \frac{1}{2}$

### 21. REVIEW QUESTION 10

Pick a number consisting of not more than six digits.

What is the probability of a 9 being at least one of the digits?

## 22. REVIEW QUESTION 10

$$\begin{aligned} & P(\text{at least one } 9) \\ &= 1 - P(\text{no digit is a } 9) \\ &= 1 - \left(\frac{9}{10}\right)^6 = .4686 \end{aligned}$$