Math 210 Lecture Notes: Chapters 1 and 2 Review

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1. Review Question 1

Find the y-intercept of the line $3x + y = 4$:

(a) 1.
(b) 2.
(c) 3.
(d) 4.
(e) None of the above.

2. Question 1 Solution

To find the y-intercept, plug $x = 0$ into $3x + y = 4$:

$0 \cdot x + y = 4$

or $y = 4$.

Answer (d)

3. Review Question 2

Find the point of intersection of the lines:

$$2x - 3y = -8 \quad \text{and} \quad 4x - y = 2$$

(a) $\left(\frac{7}{5}, \frac{-18}{5}\right)$.

(b) $\left(\frac{7}{5}, \frac{18}{5}\right)$.

(c) $\left(\frac{1}{5}, \frac{14}{5}\right)$.

(d) $(-4, 0)$.

(e) None of the above.

4. Question 2 Solution

$-2\times$ Equation 1: $-4x + 6y = 16$

Equation 2: $4x - y = 2$

Add: $0x + 5y = 18$

So $y = \frac{18}{5}$

and $2x = 3y - 8 = 3\frac{18}{5} - 8 = \frac{54}{5} - \frac{40}{5} = \frac{14}{5}$

Dividing by 2, $x = \frac{7}{5}$

Answer (b)

5. Review Question 3

Given:

$$\begin{bmatrix} 1 & 0 & -2 & -2 \\ 0 & 1 & 0 & -5 \\ -4 & 0 & 9 & 9 \\ 0 & 2 & 1 & -8 \end{bmatrix}^{-1} = \begin{bmatrix} 9 & 0 & 2 & 0 \\ -20 & -9 & -5 & 5 \\ 8 & 2 & 2 & -1 \\ -4 & -2 & -1 & 1 \end{bmatrix}$$

solve

$$\begin{array}{rrrrrrrr} w & & - & 2y & - & 2z & = & 4 \\ & x & & & - & 5z & = & 3 \\ -4w & & + & 9y & + & 9z & = & 2 \\ & 2x & + & y & - & 8z & = & 1 \end{array}$$

6. Question 3 Answers

In the solution:

(a) $z = -23$
(b) $z = -15$
(c) $z = -17$
(d) $z = -19$
(e) None of the above.

7. Question 3 Solution

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = A^{-1} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 2 & 0 \\ -20 & -9 & -5 & 5 \\ 8 & 2 & 2 & -1 \\ -4 & -2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

So $z = \begin{bmatrix} -4 & -2 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$

$= -16 - 6 - 2 + 1 = -23$

Answer (a)

8. Review Question 4

Consider the inverse (if it exists) of:

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 3 & 2 & -2 \end{bmatrix}$$

(a) The entry in the second row and third column is 0.
(b) The entry in the third row and third column is 1/4.
(c) The entry in the third row and third column is -1/4.
(d) The entry in the third row and third column is 1.
(e) The matrix is not invertible.

9. Question 4 Solution

$$\begin{bmatrix} 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 3 & 2 & -2 & 0 & 0 & 1 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 2 & 1 & 1 & 1 & 0 & 0 \\ 3 & 2 & -2 & 0 & 0 & 1 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 & -2 & 0 \\ 0 & -1 & -5 & 0 & -3 & 1 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 & 2 & 0 \\ 0 & -1 & -5 & 0 & -3 & 1 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 0 & 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 & 2 & 0 \\ 0 & 0 & -4 & -1 & -1 & 1 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 0 & 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 & 2 & 0 \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{4} & \boxed{-\frac{1}{4}} \end{bmatrix}$$

Answer (b)

10. Review Question 5

Let

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 7 & 0 & -2 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 2 \\ 1 & -1 \\ 3 & 4 \\ -2 & 0 \end{bmatrix}$$

If possible,find the entry in the first row and second column of AB.

(a) -14
(b) 6
(c) 12

(d)13
(e) The product is undefined.

11. Question 5 Solution

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -1 \\ 4 \\ 0 \end{bmatrix} = 2 - 2 + 12 + 0 = 12$$

Answer (c)

12. Review Question 6

Which of the following matrices are invertible?

$$A = \begin{bmatrix} 8 & -2 \\ -16 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 5 \\ -1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 2 \\ 0 & 4 \end{bmatrix}, \quad D = \begin{bmatrix} 8 & 4 \\ -16 & 8 \end{bmatrix}$$

(a) A only.
(b) A and B only.
(c) A,B,C and D.
(d) B and D only.
(e) Some other selection.

13. Question 6 Solution

A 2 by 2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ fails to be invertible if and only if $ad - bc = 0$

Matrix	$ad - bc$
A	$32 - 32 = 0$
B	$0 - (-5) = 5$
C	$0 - 0 = 0$
D	$64 - (-64) = 128$

The invertible matrices are B and D

Answer (d)

14. Review Question 7

Solve the system:

$$\begin{aligned} 2x + y + 2z &= 0 \\ 3y + 6z &= -18 \\ y + 2z &= 4 \end{aligned}$$

In the solution:

(a) $x = 2$.
(b) $x =$any value.
(c) $x = -7$.
(d) No solution.
(e) None of the above.

15. Question 7 Solution

$$\begin{bmatrix} 2 & 1 & 2 & 0 \\ 0 & 3 & 6 & -18 \\ 0 & 1 & 2 & 4 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 2 & 1 & 2 & 0 \\ 0 & 1 & 2 & 4 \\ 0 & 3 & 6 & -18 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 2 & 1 & 2 & 0 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & -30 \end{bmatrix}$$

The system is inconsistent.

Answer (d)

16. Review Question 8

Determine the value of k so that the following system has infinitely many solutions.

$$\begin{cases} 4x + 2y &= 5 \\ 12x + ky &= 15 \end{cases}$$

(a) -10
(b) 6
(c) 0
(d) -6
(e) None of the above.

17. Question 8 Solution

$$\begin{bmatrix} 4 & 2 & 5 \\ 12 & k & 15 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 4 & 2 & 5 \\ 0 & k-6 & 0 \end{bmatrix}$$

The system has infinitely many solutions when $k - 6 = 0$, or when $k = 6$.

Answer (b)

18. Review Question 9

If A is a 5x2 matrix and the matrix product ACC is defined, what is the size of C?

(a) 2x2.
(b) 5x2.
(c) 5x5.
(d) 2x5.
(e) None of the above.

19. Qustion 9 Solution

Since CC is defined, the number of columns of C must equal the number of rows of C, that is, C must be a square matrix.

Since AC is defined, the number of row of C must equal the number of columns of $A = 2$.

dimension of C is 2×2

Answer (a)

20. Review Question 10

Suppose that you performed Gauss-Jordan elimination to solve a system of equations with variables x,y and z. You ended up with the augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Which of the following statements is true?

21. Question 10 Answers

(a) There are no solutions to the system.
(b) The general solution is:

$$\begin{aligned} x &= -6 \\ y &= 4 \\ z &= \text{any value} \end{aligned}$$

(c) A specific (particular) solution is

$$x = 6 \qquad y = -4 \qquad z = 0$$

(d) A specific (particular) solution is

$$x = 6 \qquad y = 4 \qquad z = 5$$

(e) None of the above.

22. Question 10 Solution

The general solution to the system is

$x = 6$

$y = -4$

$z = z =$ anything

Answer (c)

23. Review Question 11

Which statements are true?

[I] A non-square matrix never has an inverse.
[II] If A is a matrix then A+A always exists.
[III] If A is a matrix and c a scalar then cA always exists.
[IV] If A and B are matrices then A-B always exists.

(a) I, II and III only.
(b) III and IV only.
(c) III only.
(d) II and III only.
(e) Some other selection.

24. Question 11 Solution

[I] A non-square matrix never has an inverse. True

[II] If A is a matrix then A+A always exists. True

[III] If A is a matrix and c a scalar then cA always exists. True

[IV] If A and B are matrices then A-B always exists. False

Answer (a)

25. Review Question 12

Let

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 4 \\ 0 & -1 \end{bmatrix}$$

Compute the top right entry of $AB - BA$

(a) 4.
(b) 3
(c) 2.
(d) 1.
(e) 0.

26. Question 12 Solution

The top right entry of AB is

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -1 \end{bmatrix} = 4 - 2 = 2$$

The top right entry of BA is

$$\begin{bmatrix} -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = -2 + 4 = 2$$

The top right entry of $AB - BA$ is $2 - 2 = 0$

Answer (e)

27. Review Question 13

Let

$$A = \begin{bmatrix} 3 & 5 & 1 \\ -2 & -4 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -2 & -4 \\ -3 & 1 & -5 \end{bmatrix}$$

Which of the following statements about $(A - 2B)$ are true?

[I] Its size is 2x3.
[II] The entry in the first row, first column is 3.
[III] The entry in the first row, second column is 1.

(a) I only.
(b) I and II only.
(c) III only.
(d) II only.
(e) II and III only.

28. Question 13 Solution

$$A-2B=\begin{bmatrix}3 & 5 & 1\\ -2 & -4 & 2\end{bmatrix}-2\begin{bmatrix}0 & -2 & -4\\ -3 & 1 & -5\end{bmatrix}$$

$$A-2B=\begin{bmatrix}3 & 5 & 1\\ -2 & -4 & 2\end{bmatrix}-\begin{bmatrix}0 & -4 & -8\\ -6 & 2 & -10\end{bmatrix}$$

$$A-2B=\begin{bmatrix}3 & 9 & 9\\ 4 & -6 & 12\end{bmatrix}$$

Statements I and II are correct.

Answer (b)

29. Review Question 14

Which of the following systems have no solutions?

$$I.\begin{cases} x+2y-4z &= 10\\ y+z &= -6\end{cases} \qquad II.\begin{cases} x+2y-4z &= 10\\ y+z &= -6\\ -y-z &= -6\end{cases}$$

$$III.\begin{cases} x+2y-4z &= 10\\ y+z &= -6\\ w &= 8\end{cases}$$

(a) I only.
(b) I and II only.
(c) II only.

(d) III only.
(e) Some other selection.

30. Question 14 Solution

System II reduces as follows

$$\begin{bmatrix} 1 & 2 & -4 & 10 \\ 0 & 1 & 1 & -6 \\ 0 & -1 & -1 & -6 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 2 & -4 & 10 \\ 0 & 1 & 1 & -6 \\ 0 & 0 & 0 & -12 \end{bmatrix}$$

System II is inconsistent.

The other two are okay.

Answer (c)