1. Review Question 1

Find the y-intercept of the line $3x + y = 4$:

(a) 1.
(b) 2.
(c) 3.
(d) 4.
(e) None of the above.

2. Question 1 Solution

To find the y-intercept, plug $x = 0$ into $3x + y = 4$:

$0 \cdot x + y = 4$

or $y = 4$.

Answer (d)

3. Review Question 2

Find the point of intersection of the lines:

$2x - 3y = -8$ and $4x - y = 2$

(a) $\left( \frac{7}{5}, -\frac{18}{5} \right)$.

(b) $\left( \frac{7}{5}, \frac{18}{5} \right)$.
(c) \( \left( \frac{1}{5}, \frac{14}{5} \right) \).

(d) \((-4, 0)\).

(e) None of the above.

4. Question 2 Solution

\(-2 \times \) Equation 1: \(-4x + 6y = 16\)

Equation 2: \(4x - y = 2\)

Add: \(0x + 5y = 18\)

So \(y = \frac{18}{5}\)

and \(2x = 3y - 8 = 3\frac{18}{5} - 8 = \frac{54}{5} - \frac{40}{5} = \frac{14}{5}\)

Dividing by 2, \(x = \frac{7}{5}\)

Answer (b)

5. Review Question 3

Given:

\[
\begin{bmatrix}
1 & 0 & -2 & -2 \\
0 & 1 & 0 & -5 \\
-4 & 0 & 9 & 9 \\
0 & 2 & 1 & -8
\end{bmatrix}^{-1} = \begin{bmatrix}
9 & 0 & 2 & 0 \\
-20 & -9 & -5 & 5 \\
8 & 2 & 2 & -1 \\
-4 & -2 & -1 & 1
\end{bmatrix}
\]

solve

\[
\begin{align*}
w & - 2y - 2z = 4 \\
x & - 5z = 3 \\
-4w & + 9y + 9z = 2 \\
2x & + y - 8z = 1
\end{align*}
\]
6. Question 3 Answers

In the solution:
(a) \( z = -23 \)
(b) \( z = -15 \)
(c) \( z = -17 \)
(d) \( z = -19 \)
(e) None of the above.

7. Question 3 Solution

\[
\begin{bmatrix}
w \\
x \\
y \\
z
\end{bmatrix} = A^{-1} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 2 & 0 \\ -20 & -9 & -5 & 5 \\ 8 & 2 & 2 & -1 \\ -4 & -2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}
\]

So \( z = \begin{bmatrix} -4 & -2 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} = -16 - 6 - 2 + 1 = -23 \)

Answer (a)

8. Review Question 4

Consider the inverse (if it exists) of:
\[
\begin{bmatrix}
2 & 1 & 1 \\
1 & 1 & 1 \\
3 & 2 & -2
\end{bmatrix}
\]

(a) The entry in the second row and third column is 0.
(b) The entry in the third row and third column is 1/4.
(c) The entry in the third row and third column is -1/4.
(d) The entry in the third row and third column is 1.
(e) The matrix is not invertible.
9. Question 4 Solution

\[
\begin{bmatrix}
2 & 1 & 1 & 0 & 0 \\
1 & 1 & 1 & 0 & 1 \\
3 & 2 & -2 & 0 & 0 & 1
\end{bmatrix}
\]

\[\rightarrow \begin{bmatrix}
1 & 1 & 1 & 0 & 1 & 0 \\
2 & 1 & 1 & 1 & 0 & 0 \\
3 & 2 & -2 & 0 & 0 & 1
\end{bmatrix}
\]

\[\rightarrow \begin{bmatrix}
1 & 1 & 1 & 0 & 1 & 0 \\
0 & -1 & -1 & 1 & -2 & 0 \\
0 & -1 & -5 & 0 & -3 & 1
\end{bmatrix}
\]

\[\rightarrow \begin{bmatrix}
1 & 1 & 1 & 0 & 1 & 0 \\
0 & 1 & 1 & -1 & 2 & 0 \\
0 & -1 & -5 & 0 & -3 & 1
\end{bmatrix}
\]

\[\rightarrow \begin{bmatrix}
1 & 0 & 1 & 1 & -1 & 0 \\
0 & 1 & 1 & -1 & 2 & 0 \\
0 & 0 & -4 & -1 & -1 & 1
\end{bmatrix}
\]

\[\rightarrow \begin{bmatrix}
1 & 0 & 1 & 1 & -1 & 0 \\
0 & 1 & 1 & -1 & 2 & 0 \\
0 & 0 & 1 & \frac{1}{4} & \frac{1}{4} & \frac{-1}{4}
\end{bmatrix}
\]

Answer (b)

10. Review Question 5

Let

\[
A = \begin{bmatrix}
1 & 2 & 3 & -1 \\
7 & 0 & -2 & 4
\end{bmatrix}
\quad \text{and} \quad
B = \begin{bmatrix}
0 & 2 \\
1 & -1 \\
3 & 4 \\
-2 & 0
\end{bmatrix}
\]

If possible, find the entry in the first row and second column of \(AB\).

(a) -14
(b) 6
(c) 12
(d) The product is undefined.

11. Question 5 Solution

\[ A = \begin{bmatrix} 1 & 2 & 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -1 \\ 4 \\ 0 \end{bmatrix} = 2 - 2 + 12 + 0 = 12 \]

Answer (c)

12. Review Question 6

Which of the following matrices are invertible?

\[ A = \begin{bmatrix} 8 & -2 \\ -16 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 5 \\ -1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 2 \\ 0 & 4 \end{bmatrix}, \quad D = \begin{bmatrix} 8 & 4 \\ -16 & 8 \end{bmatrix} \]

(a) A only.
(b) A and B only.
(c) A,B,C and D.
(d) B and D only.
(e) Some other selection.

13. Question 6 Solution

A 2 by 2 matrix \( \begin{bmatrix} a & b \\ c & d \end{bmatrix} \) fails to be invertible if and only if \( ad - bc = 0 \)

Matrix \quad ad - bc
\begin{align*}
A & \quad 32 - 32 = 0 \\
B & \quad 0 - (-5) = 5 \\
C & \quad 0 - 0 = 0 \\
D & \quad 64 - (-64) = 128
\end{align*}

The invertible matrices are \( B \) and \( D \)

Answer (d)
14. Review Question 7

Solve the system:

\[
\begin{align*}
2x + y + 2z &= 0 \\
3y + 6z &= -18 \\
y + 2z &= 4
\end{align*}
\]

In the solution:
(a) \( x = 2 \).
(b) \( x = \) any value.
(c) \( x = -7 \).
(d) No solution.
(e) None of the above.

15. Question 7 Solution

\[
\begin{pmatrix}
2 & 1 & 2 & 0 \\
0 & 3 & 6 & -18 \\
0 & 1 & 2 & 4
\end{pmatrix}
\rightarrow
\begin{pmatrix}
2 & 1 & 2 & 0 \\
0 & 1 & 2 & 4 \\
0 & 3 & 6 & -18
\end{pmatrix}
\rightarrow
\begin{pmatrix}
2 & 1 & 2 & 0 \\
0 & 1 & 2 & 4 \\
0 & 0 & 0 & -30
\end{pmatrix}
\]

The system is inconsistent.

Answer (d)
16. **Review Question 8**

Determine the value of $k$ so that the following system has infinitely many solutions.

\[
\begin{align*}
4x + 2y & = 5 \\
12x + ky & = 15 
\end{align*}
\]

(a) -10  
(b) 6  
(c) 0  
(d) -6  
(e) None of the above.

17. **Question 8 Solution**

\[
\begin{bmatrix}
4 & 2 & 5 \\
12 & k & 15
\end{bmatrix}
\rightarrow
\begin{bmatrix}
4 & 2 & 5 \\
0 & k - 6 & 0
\end{bmatrix}
\]

The system has infinitely many solutions when $k - 6 = 0$, or when $k = 6$.

Answer (b)

18. **Review Question 9**

If $A$ is a 5x2 matrix and the matrix product $ACC$ is defined, what is the size of $C$?

(a) 2x2.  
(b) 5x2.  
(c) 5x5.  
(d) 2x5.  
(e) None of the above.
19. Question 9 Solution

Since $CC$ is defined, the number of columns of $C$ must equal the number of rows of $C$, that is, $C$ must be a square matrix.

Since $AC$ is defined, the number of row of $C$ must equal the number of columns of $A = 2$.

dimension of $C$ is $2 \times 2$

Answer (a)

20. Review Question 10

Suppose that you performed Gauss-Jordan elimination to solve a system of equations with variables $x, y$ and $z$. You ended up with the augmented matrix:

\[
\begin{bmatrix}
1 & 0 & 0 & | & 6 \\
0 & 1 & 0 & | & -4 \\
0 & 0 & 0 & | & 0
\end{bmatrix}
\]

Which of the following statements is true?

21. Question 10 Answers

(a) There are no solutions to the system.
(b) The general solution is:
\[
\begin{align*}
x &= -6 \\
y &= 4 \\
z &= \text{any value}
\end{align*}
\]
(c) A specific (particular) solution is
\[
\begin{align*}
x &= 6 & y &= -4 & z &= 0
\end{align*}
\]
(d) A specific (particular) solution is
\[
\begin{align*}
x &= 6 & y &= 4 & z &= 5
\end{align*}
\]
(e) None of the above.
22. Question 10 Solution

The general solution to the system is

\[ x = 6 \]
\[ y = -4 \]
\[ z = z = \text{anything} \]

Answer (c)

23. Review Question 11

Which statements are true?

[I] A non-square matrix never has an inverse.
[II] If A is a matrix then A+A always exists.
[III] If A is a matrix and c a scalar then cA always exists.
[IV] If A and B are matrices then A-B always exists.

(a) I, II and III only.
(b) III and IV only.
(c) III only.
(d) II and III only.
(e) Some other selection.

24. Question 11 Solution

[I] A non-square matrix never has an inverse. True
[II] If A is a matrix then A+A always exists. True
[III] If A is a matrix and c a scalar then cA always exists. True
[IV] If A and B are matrices then A-B always exists. False

Answer (a)
25. Review Question 12

Let

\[ A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 4 \\ 0 & -1 \end{bmatrix} \]

Compute the top right entry of \( AB - BA \)

(a) 4.
(b) 3
(c) 2.
(d) 1.
(e) 0.

26. Question 12 Solution

The top right entry of \( AB \) is

\[ \begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -1 \end{bmatrix} = 4 - 2 = 2 \]

The top right entry of \( BA \) is

\[ \begin{bmatrix} -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = -2 + 4 = 2 \]

The top right entry of \( AB - BA \) is \( 2 - 2 = 0 \)

Answer (e)

27. Review Question 13

Let

\[ A = \begin{bmatrix} 3 & 5 & 1 \\ -2 & -4 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -2 & -4 \\ -3 & 1 & -5 \end{bmatrix} \]

Which of the following statements about \((A - 2B)\) are true?
[I] Its size is 2x3.
[II] The entry in the first row, first column is 3.
[III] The entry in the first row, second column is 1.

(a) I only.
(b) I and II only.
(c) III only.
(d) II only.
(e) II and III only.

28. Question 13 Solution

\[
A - 2B = \begin{bmatrix} 3 & 5 & 1 \\ -2 & -4 & 2 \end{bmatrix} - 2 \begin{bmatrix} 0 & -2 & -4 \\ -3 & 1 & -5 \end{bmatrix} \\
A - 2B = \begin{bmatrix} 3 & 5 & 1 \\ -2 & -4 & 2 \end{bmatrix} - \begin{bmatrix} 0 & -4 & -8 \\ -6 & 2 & -10 \end{bmatrix} \\
A - 2B = \begin{bmatrix} 3 & 9 & 9 \\ 4 & -6 & 12 \end{bmatrix}
\]

Statements I and II are correct.

Answer (b)

29. Review Question 14

Which of the following systems have no solutions?

\[
I. \begin{cases} x + 2y - 4z = 10 \\ y + z = -6 \end{cases} \quad II. \begin{cases} x + 2y - 4z = 10 \\ y + z = -6 \\ -y - z = -6 \end{cases} \\
III. \begin{cases} x + 2y - 4z = 10 \\ y + z = -6 \\ w = 8 \end{cases}
\]

(a) I only.
(b) I and II only.
(c) II only.
(d) III only.
(e) Some other selection.

30. **Question 14 Solution**

System II reduces as follows

\[
\begin{bmatrix}
1 & 2 & -4 & 10 \\
0 & 1 & 1 & -6 \\
0 & -1 & -1 & -6 \\
\end{bmatrix}
\]

\[
\rightarrow \begin{bmatrix}
1 & 2 & -4 & 10 \\
0 & 1 & 1 & -6 \\
0 & 0 & 0 & -12 \\
\end{bmatrix}
\]

System II is inconsistent.
The other two are okay.

Answer (c)