Math 210 Lecture Notes: Test 3 Review Chapters 3 and 4 but NOT Section 5.1

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1. Review Questions 1 and 2

For the next two problems consider the following linear programming problem:
Maximize $P = 15x + 12y$ subject to the constraints:

$$\begin{aligned} 5x + 4y &\leq 20 \\ x, y &\geq 0 \end{aligned}$$

The feasible set has three corner points: $(0, 0)$ and the intercepts of the line $5x + 4y = 20$: $(0, 5)$ and $(4, 0)$.

x	y	$P = 15x + 12y$
0	0	0
0	5	60
4	0	60

2. Review Question 1.

[1] The maximum occurs for :

(a) At $x = 0, y = 5$ and $x = 4, y = 0$ ONLY.
(b) $x = 20, y = 20$ ONLY.
(c) Along the line segment joining $x = 0, y = 5$ to $x = 4, y = 0$.
(d) Along the line segment joining $x = 0, y = 5$ to $x = 20, y = 0$.
(e) Some other value of x and y.

Since the maximum occurs at both corner points $(0, 5)$ and $(4, 0)$, this value is maintained on the entire line.

Answer: **c**

3. Review Question 2.

[2] The maximum is :

(a) 10.
(b) 20.
(c) 40.
(d) 60.
(e) None of the above.

Answer: **d**

4. Review Question 3.

Find the value of the pivot element in the tableau:

$$\left(\begin{array}{cccccccc|c} 1 & 0 & -1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 2 & 0 & -2 & 0 & 1 & 0 & 2 & 0 & -4 \\ 4 & 1 & 0 & 0 & 0 & 0 & -3 & 0 & 1 \\ -3 & 0 & 5 & 0 & 0 & 1 & -6 & 0 & 5 \\ \hline -6 & 0 & 3 & 0 & 0 & 0 & 2 & 1 & 4 \end{array}\right)$$

(a) 3. (b) -2. (c) 2. (d) -1. (e) 5.

We must resolve the -4 in row 2 of the constant column before looking for negatives in the bottom row. The only other negative in the second row is -2 in column 3. So the pivot column is 3. The only positive allowable ratio is 5/5 using the 5 in row 4.

Pivot at row 4, col 3. Answer **e**

5. Review Questions 4 and 5.

For the next two problems consider the following linear programming problem:

Maximize $P = 2x + 3y + 2z$ subject to the constraints:

$$\begin{aligned} 2x + y + 2z &\leq 26 \\ x + y - 3z &\leq 16 \\ x, y, z &\geq 0 \end{aligned}$$

The simplex tableau is

x	y	z	u	v	P	const
2	1	2	1	0	0	26
1	1	-3	0	1	0	16
-2	-3	-2	0	0	1	0

6. Review Questions 4 and 5.

x	y	z	u	v	P	const
2	1	2	1	0	0	26
1	$\boxed{1}$	-3	0	1	0	16
-2	-3	-2	0	0	1	0

$\Longrightarrow$

x	y	z	u	v	P	const
1	0	$\boxed{5}$	1	-1	0	10
1	1	-3	0	1	0	16
1	0	-11	0	3	1	48

7. Review Questions 4 and 5.

$\Longrightarrow$

x	y	z	u	v	P	const
1/5	0	$\boxed{1}$	1/5	$-1/5$	0	2
1	1	-3	0	1	0	16
1	0	-11	0	3	1	48

$\Longrightarrow$

x	y	z	u	v	P	const
1/5	0	1	1/5	$-1/5$	0	2
8/5	1	0	3/5	2/5	0	22
16/5	0	0	11/5	4/5	1	70

$\Longrightarrow x = 0,\ y = 22,\ z = 2,\ P = 70$

8. Review Questions 4 and 5.

[4] The maximum is:

(a) 5. (b) 15. (c) 24. (d) 70. (e) None of the above

Answer **d**

[5] The optimal solution is:

(a) $x = 5,\ y = 8,\ z = 0.$
(b) $x = 0,\ y = 22,\ z = 2.$
(c) $x = 0,\ y = 22,\ z = 6.$
(d) $x = 1,\ y = 11,\ z = 12.$
(e) None of the above.

Answer **b**

9. Review Questions 6 and 7.

For the next two problems:

Minimize $C = -10x + 14y - 2z$ subject to the constraints:

$$\begin{aligned} 2x + y + z &\leq 2 \\ x, y, z &\geq 0 \end{aligned}$$

Since the inequality is $\leq$, we do not use the dual.

Instead maximize $P = -C = 10x - 14y + 2z$

x	y	z	u	P	const
2	1	1	1	0	2
-10	14	-2	0	1	0

10. Review Questions 6 and 7.

x	y	z	u	P	const
2	1	1	1	0	2
-10	14	-2	0	1	0

$\Longrightarrow$

x	y	z	u	P	const
1	1/2	1/2	1/2	0	1
0	19	3	5	1	10

Since y and z are non-basic variables, $y = 0$, $z = 0$, $x = 1$, and $P = 10$.

11. Review Questions 6 and 7.

[6] The minimum is:

(a) -5.
(b) -10.
(c) 5.
(d) 10.
(e) None of the above.

Answer **b** since $C = -P$

[7] The optimal solution is:

(a) $x = 1, \ y = 0, \ z = 0.$
(b) $x = 0, \ y = 0, \ z = 2.$
(c) $x = 2, \ y = 0, \ z = 0.$
(d) $x = 0, \ y = 1, \ z = 0.$
(e) $x = 0, \ y = 2, \ z = 0.$

Answer **a**

12. Review Questions 8 and 9.

For the next two problems consider the following linear programming problem:

Maximize $P = 3x - 7y + z$ subject to the constraints:

$$\begin{aligned} x - 2y + 3z &\leq -90 \\ x + 2y + z &\leq 120 \\ x, y, z &\geq 0 \end{aligned}$$

The simplex tableau is

x	y	z	u	v	P	const
1	−2	3	1	0	0	−90
1	2	1	0	1	0	120
−3	7	−1	0	0	1	0

13. Review Questions 8 and 9.

x	y	z	u	v	P	const
1	**−2**	3	1	0	0	−90
1	2	1	0	1	0	120
−3	7	−1	0	0	1	0

$\Longrightarrow$

x	y	z	u	v	P	const
−1/2	1	−3/2	−1/2	0	0	45
2	0	4	1	1	0	30
1/2	0	19/2	7/2	0	1	−315

Since x and z are non-basic variables, $x = 0$, $z = 0$, $y = 45$, and $P = -315$

14. Review Questions 8 and 9.

[**8**] The maximum is:

(a) 315.
(b) -108.
(c) -315.
(d) -36.
(e) None of the above.

Answer **c**

[9] The optimal solution is:

(a) $x = 6,\ y = 0,\ z = 0.$
(b) $x = 0,\ y = 18,\ z = 0.$
(c) $x = 6,\ y = 45,\ z = 0.$
(d) $x = 0,\ y = 45,\ z = 0.$
(e) None of the above.

Answer **d** since x and z are non-basic variables.

15. Review Questions 10 – 12.

For the next three problems:

Mom's Old-Fashioned Casseroles produces a luncheon casserole that has 50% carbohydrates, 30% protein and 20% fat and costs $8.00 per pound. The dinner casserole has 75% carbohydrates, 20% protein and 5% fat and costs $10.00 per pound. How much of each should be produced to provide at least 3 pounds of carbohydrates, at least 1.5 pounds of protein and at most 0.5 pounds of fat at minimum cost? [HINT: Use the graphical method.]

16. Review Questions 10 – 12.

Organizing the Data:

Category	Lunch	Dinner	Requirement
Carb	.5	.75	at least 3
Protein	.3	.20	at least 1.5
Fat	.2	.05	at most .5
Cost	8	10	C

Nutrition Inequalities:

- Carb $.5x + .75y \geq 3$
- Protein $.3x + .2y \geq 1.5$
- Fat $.2x + .05y \leq .5$

Cost Equation:

- Cost $C = 8x + 10y$

17. Review Questions 10 – 12.

Carb. $.5x + .75y \geq 3$

Protein. $.3x + .2y \geq 1.5$

Fat. $.2x + .05y \leq .5$

Multiply Carb Equation by 4:

1. $2x + 3y \geq 12$

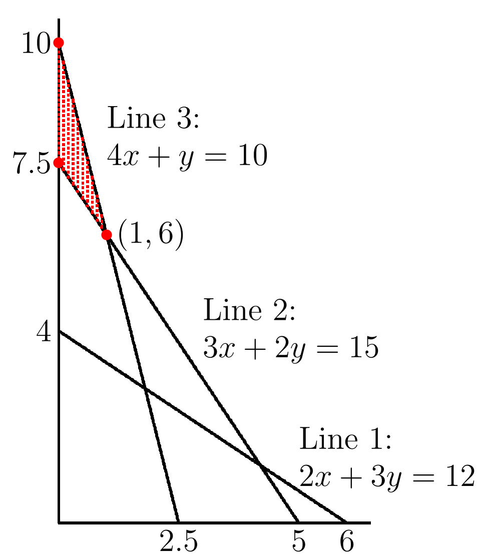
Multiply Protein Equation by 10:

2. $3x + 2y \geq 15$

Multiply Fat Equation by 20:

3. $4x + y \leq 10$

18. Feasible Region



19. Review Questions 10 – 12.

The corner points of the feasible region are:

1. The y-intercept of Line 3: $4x + y = 10$

(0,10)

2. The y-intercept of Line 2: $3x + 2y = 15$

(0,7.5)

3. The intersection of Line 2 and Line 3:

(1,6)

x	y	$C = 8x + 10y$
0	10	100
0	7.5	75
1	6	68

20. Review Questions 10 – 11.

[**10**] Amount of luncheon casserole to be produced:

(a) None.
(b) 1 pound.
(c) 1.8 pounds.
(d) 6 pounds.
(e) None of the above.

Answer **b**

[**11**] Amount of dinner casserole to be produced:

(a) None.
(b) 1 pound.
(c) 1.8 pounds.
(d) 6 pounds.
(e) None of the above.

Answer **d**

21. Review Questions 12.

[**12**] Minimum cost is:

(a) \$68.00
(b) \$34.00
(c) \$18.75
(d) \$25.00
(e) None of the above.

Answer **a**

22. Review Question 13.

[**13**] How many vertices has the feasible set for:

$$\begin{aligned} x - y &\leq 15 \\ 2x - y &\leq 15 \\ x + y &\geq 6 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

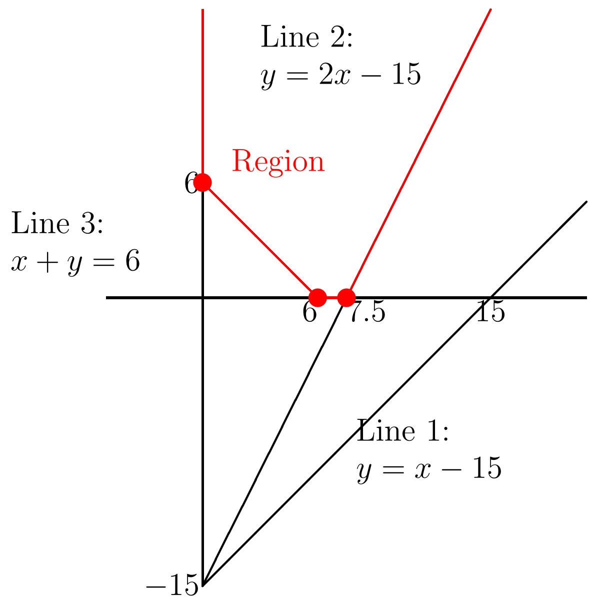
The corresponding lines are:

Line 1: $y = x - 15$ (halfplane above)

Line 2: $y = 2x - 15$ (halfplane above)

Line 3: $x + y = 6$ (halfplane above)

23. Feasible Region



24. Review Question 13.

[13] How many vertices has the feasible set for:

$$\begin{aligned} x - y &\leq 15 \\ 2x - y &\leq 15 \\ x + y &\geq 6 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

(a) 1. (b) 2. (c) 3. (d) 4. (e) 5.

There are three vertices: $(0, 6)$, $(6, 0)$, and $(7.5, 0)$

Answer **c**

25. Review Question 14.

[**14**] Is more pivoting needed in the tableau:

$$\left(\begin{array}{cccccccc|c} 1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 1 \\ 5 & 3 & 1 & 0 & 0 & 0 & 2 & 0 & 3 \\ 7 & -2 & 0 & 0 & 1 & 0 & -3 & 0 & 2 \\ 4 & 4 & 0 & 0 & 0 & 1 & -6 & 0 & 3 \\ \hline 15 & 10 & 0 & 0 & 0 & 0 & 2 & 1 & 4 \end{array}\right)$$

(a) No
(b)Yes: Pivot about a 3
(c)Yes: Pivot about a -3
(d)Yes: Pivot about a -6
(e)Yes: Pivot about a -1

No negative entry occurs in the constant column or at the bottom. Answer **a**

26. Review Question 15.

[**15**] Which of the following statements are true for the feasible set?

(A) A linear objective function might not have a maximum on it
(B) The feasible set is always convex
(C) The feasible could be empty

(a) A only
(b) B only
(c) C only
(d) B and C only
(e) A, B and C

All three statements are true

Answer **e**