Math 211 Lecture Notes: Sections 1.1 - 1.3

Richard Blecksmith

1. Math 211 Business Calculus

- Section 1
- Summer, 2009
- Mon, Tues, Wed, Thurs
- 8:00 - 9:15 a.m.
- Reavis Hall 302
- Independence Day Holiday is Thursday, July 2
- Professor Richard Blecksmith
- Dept. of Mathematical Sciences
- Northern Illinois University

2. Instructor Info

Professor Richard Blecksmith

- Office: 344 Watson Hall
- email: richard@math.niu.edu
- phone: 815-753-1835
- hours: Mon, Wed 9:30 - 11:30
- or by appointment
- webpage: http://math.niu.edu/∼richard/Math211

3. Recitation

- Teaching Assistant: Angela Antonou
- Time: 2:30 - 3:20
- Day: Tuesday
- Room: Davis Hall 309

4. Angela's Info

- Angela Antonou
- Office: 352 DuSable Hall
- email: angela@math.niu.edu
- phone: 815-753-6774
- hours: Tues 1:00 - 2:15
  Wed 12:30 - 1:45

5. Course Info

- Text: Calculus and its Applications
- 9th edition
- Bittenger and Ellenbogen
- Calculator Policy: Basic Scientific Calculators will be allowed on the exams
- You may use a graphing calculator on homework, but not on the exams

6. Test Policy

- Four Tests: Every Other Thursday
- Test Dates:
  Test 1: June 25
  Test 2: July 9
  Test 3: July 23
  Test 4: August 6
- Each Test is worth 100 points
- Recitation (homework and quizzes) 100 points

7. Grading Policy

- The lowest score of the fours tests and the recitation scores will be dropped
- Your grade will be based on the remaining four scores
- Your percentage in the class is

$$\frac{\text{Test 1} + \text{Test 2} + \text{Test 3} + \text{Test 4} + \text{Rec} - \text{lowest}}{4}$$

8. Grading Curve

- The grading curve is based on your final percentage in the class (once your lowest score is subtracted)
- The grading curve is at least as generous as
  - A 85%
  - B 75%
  - C 60%
  - D 50%

9. Test Implications

- No Final Exam
- If you miss an exam (for whatever reason) it will count as a zero and be subtacted as your lowest score
- NO MAKEUP EXAMS
- If you have an A average in the class, you need not take Test 4
- But you will need to taked the quizzes and turn in the homework for this material

10. Some Advice

- Class pace in the summer is twice as fast as during the regular semester.
- The first test is in two weeks
- Do not get behind
- Work a couple of hours every day
- Ask questions in class and in recitation

11. Homework

- The complete homework list may be found at the website
- http://math.niu.edu/math211
- Go to the section titled Homework Problems
- Note this is the same list for Spring 2009
- To get started today, the first assignment is
- Section 1.1, page 110
- Exercises 11–14, 19, 21, 27, 29, 30, 46, 57, 58

12. Limits

All of calculus is based on the concept of limit

We will spend considerable time talking about

The definition of limit

Methods for evaluating limits

13. Definition

The statement

> as x approaches the number a, the value $f(x)$ approach the limit L

means

> as x gets close to a, then $f(x)$ gets close to L

We write

$$L = \lim_{x \to a} f(x)$$

14. Example

Let $f(x) = x^2 + 3$

What is the limit as x approaches 2?

Solution: look at values of $f(x)$ for values of x near 2

x	$f(x)$
1.9	6.61
2.1	7.41
1.99	6.96015
2.01	7.0401
1.999	6.996001
2.001	7.004001

It looks like the limit is $L = 7$

15. Why the fuss

Perhaps you are thinking

- Wait a minute.
- Why all the fuss?
- Couldn't we just plug $x = 2$ into $f(x) = x^2 + 3$
- and get $L = 2^2 + 3 = 7$
- That's right.
- But not all examples are this easy.

16. Tougher Example

Let $f(x) = \dfrac{x^2 + 3x - 10}{x^2 - 4}$

What is the limit as x approaches 2?

Plugging $x = 2$ into $f(x)$ gives

$$f(2) = \frac{2^2 + 3 \cdot 2 - 10}{2^2 - 4} = \frac{4 + 6 - 10}{4 - 4} = \frac{0}{0}$$

Oops. We violated the golden rule:

YOU CANNOT DIVIDE BY ZERO

17. Finding the limit

We are forced to look at values of x near $a = 2$

x	$f(x) = \dfrac{x^2+3x-10}{x^2-4}$
1.9	1.7692
2.1	1.7317
1.99	1.75187
2.01	1.74813
1.999	1.750187
2.001	1.74981

It looks like the limit is $L = 1.75$

18. Algebra Rescue

Is there an easier way to find this limit?

Yes, factor:

$$\begin{aligned}
\lim_{x\to 2} f(x) &= \lim_{x\to 2} \frac{x^2+3x-10}{x^2-4} \\
&= \lim_{x\to 2} \frac{(x+5)(x-2)}{(x-2)(x+2)} \\
&= \lim_{x\to 2} \frac{(x+5)}{(x+2)} \\
&= \tfrac{(2+5)}{(2+2)} \\
&= \tfrac{7}{4} = 1.75
\end{aligned}$$

Notice: this is the same answer we got before.

19. Two Sided Limits

Both of the limits we considered have the same value when x approaches a

- from the left or
- from the right.

20. One Sided Limits

We write

$$L = \lim_{x \to a^+} f(x)$$

to indicate that x approaches a from the right

We write

$$L = \lim_{x \to a^-} f(x)$$

to indicate that x approaches a from the left

21. Postage Stamp Example

Let $P(x)$ be the postage required to mail a letter weighing x ounces. According to the U. S. Post Services, postage costs 44 cents for the first ounce and 17 cents for each additional ounce.

$\lim_{x \to 1^-} P(x) = \lim_{x \to 1^-} 44 = 44$

while

$\lim_{x \to 1^+} P(x) = \lim_{x \to 1^+} 44 + 17 = 61$

Notice these two limits are different.

22. Two Sided Limit Theorem

The limit $\lim_{x \to a}$ exists if and only if

the limit from the left and the limit from the right

- both exist
- and are equal.

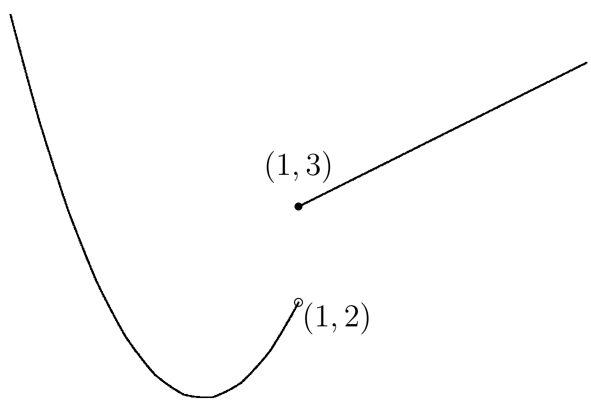
In this case, the two-sided limit is this common value:

$$\lim_{x \to a} = \lim_{x \to a^+} = \lim_{x \to a^-}$$

23. Two Way Definitions

Consider a function $f(x)$ definied by

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1 \\ (5 + x)/2 & \text{if } x \geq 1 \end{cases}$$



24. Algebraic Solution

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1 \\ (5 + x)/2 & \text{if } x \geq 1 \end{cases}$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2 + 1 = 1^2 + 1 = 2$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (5 + x)/2 = (5 + 1)/2 = 3$$

Because these two limits are unequal

$\lim_{x \to 1} f(x)$ does not exist

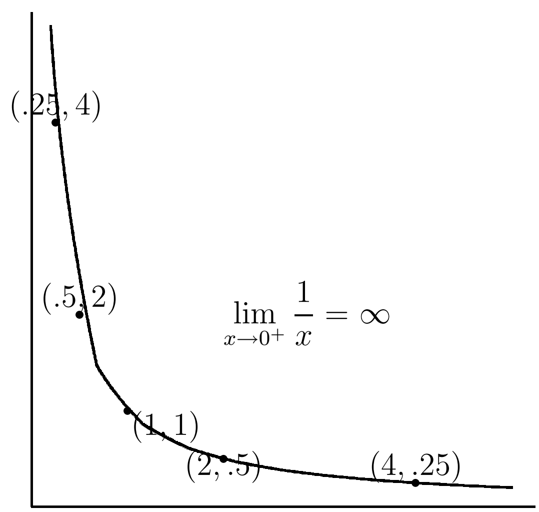
25. Infinity

What is $\lim_{x\to 0^+} \frac{1}{x}$

Makinge a chart by using values x approaching 0 from the right:

x	$f(x) = 1/x$
.1	10
.01	100
.001	1000
.0001	10000
.00001	100000
.000001	1000000

26. Graph of 1/x



27. Infinity Continued

The graph of $y = \frac{1}{x}$ reveals two interesting facts:

- The closer x gets to 0 (on the right), the larger the value of y.
- Indeed $\frac{1}{x}$ increases without bound as $x \to 0^+$
- Write: $\lim_{x\to 0^+} \frac{1}{x} = \infty$

- While as x gets large, $\frac{1}{x}$ gets small.
- Write: $\lim_{x \to \infty} \frac{1}{x} = 0$

28. Definition

The statement

$f(x)$ is continuous at the number a

means

the limit $\lim_{x \to a} f(x)$ can be computed by plugging $x = a$ into $f(x)$

That is,

$$\lim_{x \to a} f(x) = f(a)$$

29. Requirements

The definition of continuity requires

- $f(a)$ exists (that is, the function is defined at a)
- the limit $L = \lim_{x \to a} f(x)$ exists
- It is important to emphasize that L must be a real number
- L cannot be ∞ or $-\infty$
- $L = f(a)$

30. Example: No Value

Consider the function

$$y = f(x) = \frac{x^2 - 36}{x - 6}$$

Is $f(x)$ continuous at $x = 6$?

No—because the function is not defined at $x = 6$:

$f(6) = \frac{6^2 - 36}{6 - 6} = \frac{0}{0} =$ undefined

31. Second Example

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x < 2 \\ 5x - 3 & \text{if } x \geq 2 \end{cases}$$

Is $f(x)$ continuous at $x = 2$?

In this case, the function is defined at $x = 2$:
$f(2) = 5 \cdot 2 - 3 = 7$

But the limit does not exist:

- $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x^2 - 1 = 2^2 - 1 = 3$
- $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 5x - 3 = 5 \cdot 2 - 3 = 7$
- $\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x) \implies \lim_{x \to 2}$ does not exist

32. Third Example

$$f(x) = \begin{cases} x^2 - 3 & \text{if } x < 2 \\ 5x - 9 & \text{if } x \geq 2 \end{cases}$$

Is $f(x)$ continuous at $x = 2$?

In this case, the function is defined at $x = 2$:
$f(2) = 5 \cdot 2 - 9 = 1$

the limit exists:

- $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x^2 - 3 = 2^2 - 3 = 1$
- $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 5x - 9 = 5 \cdot 2 - 9 = 1$
- $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = 1 \implies \lim_{x \to 2} f(x) = 1$

$f(x)$ is continuous at $x = 2$ because $\boxed{\lim_{x \to 2} f(x) = f(2) = 1}$

33. Sunrise in Chicago March 2009

Date	Sunrise	Sunset	Length	Difference
Mar 3	6:22	5:44	11h 21m 35s	+ 8m 19s
Mar 6	6:17	5:47	11h 29m 59s	+ 8m 22s
Mar 9	7:12	6:51	11h 38m 25s	+ 8m 24s
Mar 12	7:07	6:54	11h 46m 52s	+ 8m 27s
Mar 15	7:02	6:58	11h 55m 21s	+ 8m 27s
Mar 18	6:57	7:01	12h 03m 50s	+ 8m 27s
Mar 21	6:52	7:04	12h 12m 20s	+ 8m 27s
Mar 24	6:47	7:08	12h 20m 49s	+ 8m 27s
Mar 27	6:42	7:11	12h 29m 17s	+ 2m 49s
Mar 30	6:37	7:14	12h 37m 43s	+ 8m 25s

34. Sunrise in Chicago Contnued

Notice that

The average change in the length of day between March 18 and March 21 is

$$\frac{12\text{hr}\,12\text{min}\,20\text{sec} - 12\text{hr}\,3\text{min}\,50\text{sec}}{3 \text{ days}}$$

$$= \frac{11\text{min}\,80\text{sec} - 3\text{min}\,50\text{sec}}{3 \text{ days}}$$

$$= \frac{8\text{min}\,30\text{sec}}{3 \text{ days}}$$

$$= 2\text{min}\,50\text{sec} \text{ per day}$$

This means that in the middle of March, days are getting longer by roughly 3 minutes per day.

35. Average rate of change

If $L(t)$ = the length of the day for the date t,

then average rate of change between dates a and b is

$$\frac{L(b) - L(a)}{b - a}$$

So if $a =$ March 18 and $b =$ March 21, then we have calculated the average to be

$$m = 170 \frac{\text{sec}}{\text{day}}$$

36. Predicting length of day

We could use this to predict the length of March 30.

Since March 30 is 15 days from March 15, we would expcet the length of day for March 30 to be

$L(\text{Mar } 15) + 170 \frac{\text{sec}}{\text{day}} \times 15$ days

$=$ 11hr 55min 21sec $+$ 2550 sec

$=$ 11hr 55min 21sec $+$ 42min 30 sec

$=$ 12hr 37min 51sec

only 8 seconds more than the acutal value (using the table) of 12 hr 37 min 43 sec.

Thus we could predict the acutal time of sunrise on March 30 to within 4 seconds. Why?

37. Predicting length of day

Now suppose we use this same method to predict the length of day for June 16.

The number of day March 15 to June 13 is

$$16 + 30 + 31 + 16 = 93$$

So the length of June 16 should be approximately

$L(\text{Mar } 15) + 170\frac{\text{sec}}{\text{day}} \times 93 \text{ days}$

$= 11\text{hr}\,55\text{min}\,21\text{sec} + 15810 \text{ sec}$

$= 11 \text{ hr}\,55 \text{ min}\,21 \text{ sec} + 4 \text{ hr}\,23 \text{ min}\,30 \text{ sec}$

$= 16 \text{ hr}\,18\text{min}\,51\text{sec}$

which is over an hour more than the actual value of

$= 15 \text{ hr}\,12\text{min}\,53\text{sec}$

What went wrong?

38. Data for June 2009

Date	Sunrise	Sunset	Length	Difference
Jun 11	5:15	8:26	15h 10m 37s	+ 38s
Jun 12	5:15	8:26	15h 11m 12s	+ 34s
Jun 13	5:15	8:27	15h 11m 43s	+ 30s
Jun 14	5:15	8:27	15h 12m 10s	+ 27s
Jun 15	5:15	8:28	15h 12m 33s	+ 23s
Jun 16	5:15	8:28	15h 12m 53s	+ 19s
Jun 17	5:15	8:28	15h 13m 08s	+ 15s
Jun 18	5:15	8:29	15h 13m 20s	+ 11s
Jun 19	5:15	8:29	15h 13m 28s	+ 07s
Jun 20	5:16	8:29	15h 13m 32s	+ 03s
Jun 21	5:16	8:29	15h 13m 32s	¡ 1s

39. Data for June 2009 Cont'd

Date	Sunrise	Sunset	Length	Difference
Jun 21	5:16	8:29	15h 13m 32s	¡ 1s
Jun 22	5:16	8:30	15h 13m 28s	- 03s
Jun 23	5:16	8:30	15h 13m 20s	- 07s
Jun 24	5:17	8:30	15h 13m 08s	- 11s
Jun 25	5:17	8:30	15h 12m 52s	- 15s
Jun 26	5:17	8:30	15h 12m 33s	- 19s
Jun 27	5:18	8:30	15h 12m 10s	- 23s
Jun 28	5:18	8:30	15h 11m 42s	- 27s
Jun 29	5:19	8:30	15h 11m 11s	- 30s
Jun 30	5:19	8:30	15h 10m 37s	- 34s

40. Explanation

- The assumption that the length of day increase by rougly 3 minutes per day is totally wrong for the month of June!
- Past June 6, the average daily increase is always under a minute
- It gets worse
- Starting on June 21, the lenght of the day begin to **decrease**

41. Baseball Ticket Prices

Based on data from MLB

the average price of a ticket to a major league game can be approximated by

$$p(t) = 0.03t^2 + 0.56t + 8.63$$

where t is the number of years after 1991 and $p(t)$ is in dollars

Find $p(8)$ (the price in 1999)

Answer: $p(8) = 0.03(8^2) + 0.56(8) + 8.63 = 1.92 + 4.48 + 8.63 = 15.03$

Find $p(18)$ (the price in 2009)

Answer: $p(18) = 0.03(18^2) + 0.56(18) + 8.63 = 9.72 + 10.08 + 8.63 = 28.43$

42. Average increase in ticket price

Between 1999 ($a = 8$) and 2009 ($b = 18$) The average increase in ticket price is

$$\frac{p(18) - p(8)}{18 - 8} = \frac{28.43 - 15.03}{10} = \frac{13.4}{10} = 1.34$$

According to this model, the average price of an MLB ticket has increased by an average of \$1.34 over the past ten years.

Source: data from www.teammarketing.com

43. Average speed of a car

If you drive to Chicago, a distance of 60 miles, in one hour, then your average speed for the trip is

$$\frac{60 \text{ miles}}{1 \text{ hour}} = 60 \text{ mph}$$

or $\frac{60 \text{ miles}}{60 \text{ minutes}} = 1$ mile per min

This does not mean that every minute you are travelling exactly one mile

You may

- get stuck in traffic
- slow down at a toll booth
- speed up on an open stretch
- get pulled over by the cops