Math 229 **Final Examination** **Dec 9, 2005** **Name:** ____________________

No books, notes, or calculators are allowed on this test. Your instructor will provide you with scratch paper, if you need it. Be sure that all of your work is shown and that it is well organized and legible.

Part I. Proficiency (100 points) Each problem (or part of a problem with multiple parts) is worth 5 points.

1. Determine the following limits (if they exist):

a) $\lim\limits_{t \to 1} \dfrac{t^2 + t - 2}{t^2 - 1}$

b) $\lim\limits_{x \to 0} \dfrac{5x}{\sin 2x}$

c) $\lim\limits_{x \to \infty} \dfrac{2x^3 + 7}{6 + x - x^3}$

2. Determine where the function $f(x) = \begin{cases} \sqrt{-x} & x < 0 \\ 3 - x & 0 \le x \le 3 \\ (x-3)^2 & x > 3 \end{cases}$ is discontinuous and explain why.

3. Find the derivatives of each of the following functions. You do not need to simplify your answers.

   a) $f(x) = x^3 + \sec x - \sqrt{x-1}$

   b) $f(x) = (x^2+3)^3(2x^3 - x + 1)^5$

   c) $s(t) = \cos^4(1-t)$

   d) $y = \dfrac{x^3+1}{\sqrt{x-1}}$

4. State the Intermediate Value Theorem and use it to show that the equation $x^3 - 15x + 1 = 0$ has at least one solution in the interval $[0, 1]$

5. Use implicit differentiation to find $\frac{dy}{dx}$ for $xy + 2x + 3y = 1$.

6. Find the horizontal and vertical asymptotes of the curve $y = \dfrac{3x+1}{2x-5}$.

7. Find the absolute maximum and minimum values of the function $g(x) = \sqrt{4 - x^2}$ on the interval $-2 \leq x \leq 1$.

8. Determine the interval(s) on which the function $h(x) = x^4 + 2x^3 - 6$ is increasing.

9. Determine where the function $k(x) = 1 - \cos x$ is concave up over the interval $[0, 2\pi]$.

10. The derivative of a function, $f'(x)$, is negative everywhere. We also know that $f(0) = 0$. What must be true about $f(-1)$? Explain your answer.
(i) $f(-1)$ is negative.
(ii) $f(-1)$ is positive.
(iii) $f(-1)$ is zero.
(iv) There is not enough information to conclude anything about $f(-1)$.

11. Find the derivative of the function $f(x) = \displaystyle\int_1^x \sin(\sqrt{1-t})\, dt$.

12. Find each of the following integrals:

a) $\displaystyle\int_1^2 \left(\frac{t^2}{2} + 2t^3\right) dt$

b) $\displaystyle\int \left(\frac{x\sqrt{x} + \sqrt{x}}{x^2}\right) dx$

c) $\displaystyle\int \cos\theta\,(\tan\theta + \sec\theta)\, d\theta$

d) $\displaystyle\int_0^1 5r\sqrt{1 - r^2}\, dr$

Part II. Applications (100 points) Each problem is worth 10 points.

13. State the limit definition of derivative and use it to find the derivative of $f(x) = \dfrac{1}{x-1}$.

14. Let $f(x) = x^2 + ax + b$. The line $y = 2x$ is tangent to the curve at the point (2,4). What are the values of a and b?

15. A rocket lifts off the surface of the earth with a constant acceleration of 20 $\mathrm{m/sec^2}$. At what speed will the rocket be going 1 minute later? How far will the rocket be above the ground at that time?

16. A function $f(x)$, whose domain is the interval $(-\infty, 8]$, has the following first and second derivatives:

$$f'(x) = \frac{16-3x}{2\sqrt{8-x}} \quad \text{and} \quad f''(x) = \frac{32-3x}{2(x-8)\sqrt{8-x}}.$$

In addition, $f(0) = f(8) = 0$ and $\lim_{x \to -\infty} f(x) = -\infty$. Sketch a plausible graph of $f(x)$, labeling the value(s) of x where this function has a local maximum, minimum, or an inflection point.

17. The volume of a cube is increasing at the rate of 10 cubic cm/sec. How fast is the length of an edge increasing when the volume is 27,000 cubic cm?

18. Find the coordinate of the point (x, y) on the curve $y = \sqrt{x}$ closest to the point $(4, 0)$.

19. Find the linearization of $f(x) = \dfrac{1}{1 + \tan x}$ at $x = 0$.

20. Show that $f(x) = x + \frac{1}{x}$ satisfies the hypotheses of the Mean Value Theorem on the interval $[\frac{1}{2}, 1]$ and find the value of c that the Mean Value Theorem guarantees.

21 Estimate $\int_0^2 (x^2 + x)\,dx$ by computing the Riemann sum using 4 rectangles, with midpoints as sample points. You can leave your answer in fractional form.

22. Find the area enclosed between the curves $y = 2\sin x$ and $y = \sin 2x$ over the interval $0 \leq x \leq \pi$.