Math 229 Exam 2 October 27, 1999

Show work. You need not simplify #1 and 2.

1. [10 pts] Find y' if $y = \dfrac{(x-1)(x-4)}{(x-2)(x-3)}$.

2. [10 pts] Find y' if $y = \sqrt{\sin\sqrt{x}}$.

3. [10 pts] Find $\frac{dy}{dx}$ if $\cos(x-y) = y\sin x$.

4. [10 pts] Find $f''(x)$ if $f(x) = 8\tan(x/2)$.

5. [10 pts] Find the following limits: (a) $\lim\limits_{x\to 0} \dfrac{\cot 2x}{\csc x}$ (b) $\lim\limits_{x\to\infty} \dfrac{5-2x}{3-x}$

6. [15 pts] Two sides of a triangle have lengths 12 and 15 m. The angle between them is increasing at a rate of 2 degrees per minute. How fast is the length of the third side increasing when the angle between the sides of fixed length is 60°?

7. [10 pts] Find the critical points of $f(x) = 2x^3 + 5x^2 - 4x$. Use the second derivative test to determine which critical points are local maximums and which are local minimums.

8. [20 pts] Consider the curve $f(x) = \dfrac{x^3-1}{x^3+1}$, whose derivatives are:

$$f'(x) = \frac{6x^2}{(x^3+1)^2} \qquad f''(x) = \frac{-24x^4+12x}{(x^3+1)^3}$$

(a) Over what intervals is the graph increasing? (b) concave up?

(c) What are the vertical asymptotes? (d) horizontal asymptotes?

(e) What are the critical points, if any? (f) inflection points, if any?

(g) Sketch the graph. Identify all vertical and horizontal asymptotes, critical points, and points of inflection.

9. [5 pts] Explain why the conclusion of the Mean Value Theorem does not hold for $f(x) = \dfrac{1}{x^2}$ on $[-1, 1]$.