Math 229 Exam 3 November 23, 1999

1. [20 pts] A Norman window has the shape of a rectangle surmounted by a semicircle. (The diameter of the semicircle is equal to the width of the rectangle.) If the perimeter of the window is 30 ft, find the dimensions of the window so that the greatest possible amount of light is admitted.

2. [20 pts] Compute the integral $\int_0^1 x^2 + 7\,dx$ by finding the limit of the sum

$$S_n = \sum_{k=1}^{n} f(x_k)\Delta x$$

of the areas of n rectangles of width Δx and height $f(x_k)$. (No credit for any other method.)

Formulas: $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$, $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$, $\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$.

3. [10 pts] Find $f(x)$ if $f''(x) = x + \sqrt{x},\ f(1) = 1,\ f'(2) = 2.$

4. [25 pts] Integrate:

(a) $\int_0^1 \frac{1}{\sqrt{x+1}}\,dx$ (b) $\int (x+1)\sin(x^2+2x+3)\,dx$

(c) $\int \left(x + \frac{1}{x}\right)^2 dx$ (d) $\int_0^{\pi/4} \sqrt{\tan x}\ \sec^2 x\,dx$

(e) $\int t^3(t^2+5)^{17}\,dt$

5. [10 pts] Evaluate the following sums: (a) $\sum_{k=1}^{n}(2k-1)$ (b) $\sum_{k=501}^{1000} k^2$.

6. [5 pts] Express the limit $\lim_{n\to\infty} \sum_{k=1}^{n} x_k \sin x_k\,\Delta x$ as a definite integral on the interval $[0, \pi]$.

7. [5 pts] Find $\frac{d}{dx}\int_0^x \frac{t^3 - \cos t}{\tan t - 7}dt.$

8. [5 pts] Explain why Newton's method doesn't work for finding the root of the equation $x^3 - 3x + 6 = 0$ if the initial approximation is chosen to be $x_1 = 1$.