

Math 230 Final Examination May 4, 2004

1. Find the volume of the solid obtained by rotating the triangle with vertices $(1, 2)$, $(1, 4)$, and $(4, 4)$ about the x -axis.

2. Let $f(x)$ be a function defined for $x > 2$ whose derivative is $f'(x) = \frac{3}{x-2}$.
 - (a) Show that $f(x)$ is one-to-one for $x > 2$, so that it has an inverse function $g(x) = f^{-1}(x)$ for $x > 2$.

 - (b) If $f(1) = 5$, compute $g'(5)$.

3. A colony of bacteria is grown under ideal conditions in a laboratory so that the population increases exponentially with time. At the end of 3 hours there are 8000 bacteria. At the end of 5 hours there are 128,000 bacteria. How many bacteria were present initially?

4. (a) For what values of x is $\sin^{-1}(\tan^{-1} x)$ defined?

(b) Compute $\frac{dy}{dx}$ for $y = \sin^{-1}(\tan^{-1} x)$.

5. Differentiate $y = e^{\sqrt{x}} \ln(x^2 + 1)$.

6. Use logarithmic differentiation to find the derivative of $y = x^{\sin x}$.

7. Find the length of the curve $y = \left(\frac{x}{3}\right)^{3/2}$ from $(3, 1)$ to $(12, 8)$.

8. (a) Show that L'Hôpital's Rule **fails** to determine the limit $\lim_{x \rightarrow \infty} \frac{x + \sin x}{x}$.

(b) Evaluate the limit in (a) without using L'Hôpital's Rule.

9. Integrate $\int \frac{e^{\tan x}}{\cos^2 x} dx$.

10. Integrate $\int \frac{\sin x}{2 \cos x + 3} dx$.

11. Evaluate the improper integral $\int_2^{\infty} \frac{dx}{x\sqrt{x^2 - 4}}$.

12. Integrate $\int x \sec^2 x dx$.

13. Integrate $\int \frac{x^2 + x + 2}{x^2 - 1} dx$.

14. Compute the limit: $\lim_{n \rightarrow \infty} \left(1 + \frac{2}{n}\right)^n$

15. Suppose you place an infinite number of non-overlapping circles on the x -axis. The first circle has radius 1, the second circle has radius $\frac{1}{2}$, and in general, the n -th circle has radius $\frac{1}{n}$.

(a) Show that the sum of the circumferences of all the circles is infinite.

(b) Show that the sum of the areas of all the circles is finite.

16. Evaluate the exact sum of the series $\sum_{n=0}^{\infty} \left[\left(\frac{1}{\sqrt{2}}\right)^n + \frac{\pi^{2n}}{10^n} \right]$

17. Determine whether the following sum converges conditionally, absolutely, or diverges:

$$\sum_{n=0}^{\infty} (-1)^n \frac{1}{n^2 + 4}$$

18. (a) Find the n th partial sum $S_n = \sum_{k=1}^n \ln\left(\frac{k+1}{k}\right)$.

(b) Using your answer in part (a), determine whether the series $\sum_{k=1}^{\infty} \ln\left(\frac{k+1}{k}\right)$ converges or diverges.

19. Determine the radius of convergence of the power series: $\sum_{n=0}^{\infty} \frac{n! n!}{(2n)!} x^n$.

20. Use the Taylor series

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots + (-1)^n \frac{1}{(2n+1)!}x^{2n+1} + \cdots$$

to integrate $\int \frac{\sin x}{x} dx$.