Math 230 Series Review Problems

I. Evaluate the following limits or give reasons why they don't exist:

$(a)\ \lim_{n\to\infty} \dfrac{3n^{1000} - n^9 + \sqrt[3]{n^{100}}}{9 - 4n^{1000} + n^{11}}$

$(b)\ \lim_{n\to\infty} \sqrt{n+1} - \sqrt{n}$

$(c)\ \lim_{n\to\infty} \dfrac{\sin n}{n}$

$(d)\ \lim_{n\to\infty} \dfrac{\sqrt[3]{n} + \sqrt[4]{n}}{\sqrt{n} + + \sqrt[5]{n}}$

$(e)\ \lim_{n\to\infty} \dfrac{n \tan^{-1} n}{3n+4}$

$(f)\ \lim_{n\to\infty} \sin^{-1}\Big(\dfrac{n}{2n+1}\Big)$

$(g)\ \lim_{n\to\infty} \Big(1 + \dfrac{3}{n}\Big)^{4/n}$

$(h)\ \lim_{n\to\infty} \Big(3 - \dfrac{4}{n}\Big)^{2/n}$

$(i)\ \lim_{n\to\infty} \dfrac{n^3 + 3^n}{n^{101} + 4^n}$

$(j)\ \lim_{n\to\infty} \dfrac{n^3}{n+1}$

$(k)\ \lim_{n\to\infty} \ln(n+1) - \ln n$

$(l)\ \lim_{n\to\infty} n^{-\frac{1}{n}}$

$(m)\ \lim_{n\to\infty} (-1)^n \Big(\dfrac{n+1}{n}\Big)$

$(n)\ \lim_{n\to\infty} 3 + \Big(\dfrac{-2}{\pi}\Big)^n$

II. Determine the convergence or divergence of the following series. In case of convergence, determine the sum it converges to.

(a) $1 - \dfrac{1}{3} + \dfrac{1}{9} - \dfrac{1}{27} + \cdots$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{5^n}{8^{n+1}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{3 + 4^n}{5^{n-1}}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1}$

(e) $\displaystyle\sum_{n=1}^{\infty} \sin\big(\frac{1}{n}\big)$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(4n+1)(4n-3)}$

III. (a) For what p is the following sum convergent: $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(1+n^2)^p}$

(b) To how many terms should we take the partial sum of $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} n^{-3/2}$ so that it is approximated to with an error of 0.01?

IV. Determine the convergence or divergence of the following, using the integral, comparison, limit comparison, ratio, and root tests. In case it is an alternating series, specify whether the convergence is absolute or conditional.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{\sqrt[3]{n}}$

(b) $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2}$

(c) $\displaystyle\sum_{n=1}^{\infty} \ln\Big(\frac{n+1}{n}\Big)$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n - 3^n}{4^n + 1}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{3n-4}{n(n+1)\sqrt{n}}$

(f) $\displaystyle\sum_{n=2}^{\infty} \frac{n^2}{n^3 - 4}$

(g) $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$

(h) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln n}$

(i) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n}$

(j) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3n}{5n-1}$

(k) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n+3}}$

(l) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$

(m) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

(n) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^n}$

(o) $\displaystyle\sum_{n=1}^{\infty} \Big(\frac{1-3n}{3+4n}\Big)^n$

(p) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n n^2}{(n+2)!}$

(q) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+3)!}{n!7^n}$