1. [Multiplying on your fingers.] Designate your thumb as 6, your index finger as 7, etcetera, with your pinky finger representing 10. To multiply any two digits from 6 to 10, touch together the corresponding fingers on your two hands. To find the 10’s digit of the product, add up all fingers on both hands from the thumb to and including the touching fingers. The unit’s digits is the product of the number of fingers on each hand between the touching fingers and pinky (count the pinky, but not the touching finger). For example, to multiply 7 × 9, touch the index finger of your left hand to the ring finger of your right hand. The 10’s digit is 2 + 4 = 6, while the unit’s digit is 3 × 1 = 3. Thus 7 × 9 = 63. Explain why this works.

2. [Four 2’s.] Write the numbers 1 thru 10 using exactly four 2’s. For example, 1 = 22 + 22, 2 = $\frac{2}{2} + \frac{2}{2}$, etc. No other digits are allowed.

3. [Pails of Water.] You have a 5 quart and a 7 quart pail. How can you use them to fetch exactly one quart of water. Change the size of the pails to 7 and 11; 5 and 8; 6 and 15.

4. [Mental Arithmetic.] Factor 999919 in your head.

5. [Word Divisible Numbers.] We will call a number $N$ word divisible if the number of letters in the English nomenclature of $N$ actually divides $N$. For example, $N = 4$, written “four”, requires four letters, and 4 divides 4; $N = 12$, written “twelve”, requires six letters, and 6 divides 12. On the other hand, $N = 5$ is written “five”, requiring four letters, and 4 does not divide 5. Hence the numbers 4 and 12 are word divisible, while 5 is not. Spaces and hyphens are not counted as letters. Two larger examples of word divisible numbers are $N = 33$ which requires 11 letters (thirty three) and $N = 100$ which requires exactly 10 letters (one hundred). Which numbers up to 100 are word divisible? Can you find two word divisible numbers in a row? If so, what is the smallest number $N$ for which $N$ and $N + 1$ are both word divisible? What happens if we write the numbers in another language, such as French or Spanish?