1. The Blue-eyed Blond-hair Puzzler

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2. Lemma

If one girl in a collection $\{G_1, G_2, \ldots, G_n\}$ blonds has blue eyes,

then all of the girls have blue eyes.

The proof uses induction on n.

3. Consequence

The Lemma easily proves the following theorem:

Theorem. All blonds have blue eyes.

All we need is to find just one blond with blue eyes and apply the lemma.

4. Induction Proofs

To prove a statement $P(n)$ by induction requires two steps:

Step 1. Show that $P(1)$ is true.

Step 2. Show that if $P(n)$ is true, then $P(n+1)$ is also true.

5. Case $n = 1$

The Lemma is clearly true when $n = 1$.

If one girl in a collection of 1 blonds has blue eyes, then clearly all the girls in that collection have blue eyes.

6. Induction Hypothesis

Assume the lemma is true for n.

Let $G_1, \ldots, G_{n+1}$ be a collection of $n+1$ blonds

where one of the girls has blue eyes.

Without loss of generality, we may assume the girl with blue eyes is G_1.

We must show that $G_1, \ldots, G_{n+1}$ all have blue eyes.

7. Use induction hypothesis

Consider the following sets:

$S_1 = \{G_1, \ldots, G_n\}$ and

$S_2 = \{G_1, \ldots, G_{n-1}, G_{n+1}\}$.

Each is a collection of n blonds, and G_1 has blue eyes.

By the induction hypothesis on set S_1,

all n girls $G_1, \ldots, G_n$ have blue eyes.

By the induction hypothesis on set S_2

all n girls $G_1, \ldots, G_{n-1}, G_{n+1}$ have blue eyes.

In particular G_{n+1} has blue eyes.

Throwing G_{n+1} in with the first n girls we see that

all $n+1$ girls $G_1, \ldots, G_{n+1}$ have blue eyes.

This completes the induction step and the proof.

Find the error!