1a. To use the method of slices, take slices across the x-axis. We want \( V = \int A(x) \, dx \), where \( A(x) \) is the cross-sectional area, and so is the area of a disk: \( A(x) = \pi r(x)^2 \). Here \( r(x) \) is the radius of the cross-section, which is the length of the line segment from the central axis (x-axis) to the outside (the graph of the cubic). That length is the distance between the endpoints, and that distance is the difference between the y-coordinates. One is at \( y = 0 \), the other at \( y = -(x - 100)^3/1000 \), so we have \( r(x) = -(x - 100)^3/1000 \), and

\[
V = \int_{0}^{100} \pi \left( \frac{-(x - 100)^3}{1000} \right)^2 \, dx
\]

The only thing you need from the graph is the fact that the curve is in the first quadrant exactly on the interval \([0,100]\), which requires only setting \( x = 0 \) or \( y = 0 \) in the cubic equation.

For shells, we need a measure of our distance away from the central axis; the y-coordinates will do, and we see in the graph that these range from \( y = 0 \) to \( y = 1000 \). So we want \( V = \int_{0}^{1000} 2\pi y h(y) \, dy \) where \( h(y) \) is the “height” (here actually the horizontal measure) of the cylinder. Well, that’s the length of the line segment along the edge of the cylinder, which is the segment from the y-axis to the graph of the cubic. That length is the distance between the endpoints, and that distance is the difference between the x-coordinates. One is at \( x = 0 \), the other at \( x = 100 - 10y^{1/3} \). So we have \( h(y) = 100 - 10y^{1/3} \), and

\[
V = \int_{0}^{1000} 2\pi y(100 - 10y^{1/3}) \, dy
\]

1b. The first integral is a little easier if you set \( u = x - 100 \), but in any case it’s just a polynomial in \( x \) (or \( u \)) and so is easy to integrate; the other integral requires only that you multiply through and get something like \( 2\pi \int (100y - 10y^{4/3}) \, dy \). In either case a little arithmetic gives \((10^8/7)\pi\).

2a. \( \ln(3x^2 + 7x + 3) = 4 \) means \( 3x^2 + 7x + 3 = e^4 \), which is a quadratic polynomial in \( x \): in the usual notation, \( a = 3, b = 7, \) and \( c = 3 - e^4 \). The solution is \( \frac{-7 \pm \sqrt{13 - 12e^4}}{6} \). I chose those numbers because one of the roots is close to \( \pi \), but it isn’t really \( \pi \). There’s a second root, too, near \(-5.475\).

2b. Rearrange the terms a bit, since e.g. \( \ln(7^{-3}) + \ln(7^3) = \ln(7^{-3} \cdot 7^3) = \ln(1) = 0 \). The others also cancel in pairs (the middle term \( \ln(7^0) \) is already zero), so the sum is zero.

2c. So \( x = e^{\ln(1 + h)} - 1 \), right? Recall \( \exp \) and \( \ln \) are inverse functions, so this is just \( (1 + h) - 1 = h = 10^{-20} \), which is positive.

3. You should probably first note that the domain of \( f \) is limited to the positive numbers \( x \). In particular, there is no \( y \)-intercept.

Next, \( f'(x) = (1/x) - (1/100) \) so \( f''(x) = -1/x^2 \), which is always negative, so the whole thing is concave down (no inflection points). There’s a critical point at \( x = 100 \), as you can see, and it’s easy to check \( f'(x) > 0 \) to the left of that and \( f'(x) < 0 \) to the right, so the function is increasing to its maximum, then decreasing. You should get a sort of parabola, squished in by the vertical axis.

By the way you can plot some points and so on and deduce that there are two \( x \)-intercepts, one at about \( x = 4848.645 \) and the other around \( x = 4.25 \times 10^{-18} \).
4a. Substitute \( u = -2x^2 - 3 \), so \( du = -4xdx \) and the integral becomes \( \int (-3/4)du/u \) which is \( (-3/4)\ln(|u|) + C = (-3/4)\ln(|-2x^2 - 3|) + C \). You really need the absolute value bars here since \(-2x^2 - 3\) is always negative.

4b. Since \( 10^x = e^{\ln(10)x^2} \), you should probably take that whole exponent for \( u \), then get \( du = \ln(10) \cdot 2xdx \), and then the integral is \( \int e^u du/(2 \ln(10)) = e^u/(2 \ln(10)) + C = 10^x/(2 \ln(10)) + C \).

4c. Since \( \cos(\pi) = -1 \), the limit will be \( \pi^4 \) plus the limit of \( \ln(u) \) as \( u \) approaches 0 (from above), which you know is \(-\infty\). (P.S.: you have to get really really close to \( \pi \) — like, within \( 10^{-21} \) — before the function even takes on negative values at all, let alone approach \(-\infty\)!

4d. The average value is \( (1/\pi) \int_0^{\pi} \sin(360x)dx = (1/\pi) (-\cos(360x)/360) |_0^{\pi} = 0 \).

5. We have \( P(t) = P_0e^{kt} \) for some unknown \( k \) and \( t \). Let’s measure time in hours since 1pm.

Then we know \( P(2) = 30000 \) and \( P(3) = 40000 \) and want \( P(0.5) \). Well, from \( 30000 = P_0e^{2k} \) and \( 40000 = P_0e^{3k} \) we may simply divide to get \( 4/3 = e^k \), so \( e^{0.5k} = \sqrt{4/3} \). Substitute back into a previous equation and get \( P_0 = 30000/e^{2k} = 30000/(4/3)^2 = 270000/16 \), so our answer is \( P(0.5) = 270000/16 \sqrt{4/3} \). You may rewrite this in several ways, e.g. \( 11250\sqrt{3} \), about 19,486.

Here are some observations about the problems.

1. Surely all students prepared for some volume computations. Note that I used only round slices (compare 6.2 Example 7), no washers with holes, no dislocated axes, no trig or exponential functions — all of which \( \text{do} \) sometimes appear on exams, and which you may wish to review for the final exam. Also some students should review sections 1.2 and 1.3 of the text!

2a. Practice 7.3* numbers 9, 10, 14, or Chapter Review #18
2b. These are the “Laws of Logarithms”, p. 446
2c. Compare 7.3* nos. 2-4

3. Compare example 9 of 7.2* and example 7 of 7.3*

4a. I wrote this specifically thinking about 7.2* #62. Later this semester we will show you how to antidifferentiate any rational function
4b. Swiped from section 7.5#42
4c. Apart from the limit laws from Calc 1, what I was checking here was that you knew \( \ln \) tends to \(-\infty\) along the \( x \)-axis.
4d. See e.g. section 6.5 #15

5. This is really a very standard problem. Compare to e.g. section 10.4 #4

6. In the end I settled for just a couple of basic questions about work to see that you understood the bouncing springs are not the only reason to take calculus.

6a. is just the boxed equation on page 395 6b. is the FTC, the boxed equation on page 338 (which you learned in Calc 1) 6c. is a differentiation problem like those of 7.3 # 34, 43, 44.