1. (15 points) Discuss the monotonicity and concavity of \( f(x) = x^4 + 4x^3 + 5 \).

2. (15 points) A rectangular garden with an area of 150 square meters is to be enclosed by a fence and divided into two parts by an internal fence parallel to the ends. What dimensions for the whole garden will require the smallest total length of fence?
3. (20 points) Sketch the graph of \( f(x) = \frac{x^3}{1 - x^2} \), given \( f'(x) = -\frac{x^2(x + \sqrt{3})(x - \sqrt{3})}{(1 - x^2)^2} \), \( f''(x) = \frac{2x(x^2 + 3)}{(1 - x^2)^3} \) and \( \sqrt{3} \approx 1.73 \). Use the back side of Page 1 for more space.

Domain:

Intercepts

Symmetries

Asymptotes

Monotonicity

Concavity

Shapes
4. (10 points) Use the limit definition to find \( \int_0^3 (x^2 - 8x + 7) \, dx \), given \( 1 + 2 + \cdots + n = n(n + 1)/2 \) and \( 1^2 + 2^2 + \cdots + n^2 = n(n + 1)(2n + 1)/6 \).

5. (5 points) Compute \( \frac{d}{dx} \int_{x^4+1}^{\tan x} \frac{dt}{t} \).

6. (5 points each) Evaluate definite integrals.

\[ \int_{-2}^3 (x^5 + 8x) \, dx \]

\[ \int_{-4}^1 (5x + 3)\sqrt{x} \, dx \]
7. (5 points each) Find antiderivatives.

\[ \int \sqrt{3 - 5x} \, dx \]

\[ \int x^3(x^4 + 2)^5 \, dx \]

\[ \int x \cos(3 - x^2) \, dx \]

\[ \int (1 + \cos x)^6 \sin x \, dx \]

\[ \int x \cot(x^2) \csc(x^2) \, dx \]