

There are two parts to this Exam:

Part I: Do 8/10 problems

Part II: Do 2/3 problems

Indicate clearly which problems you want graded

Name (Print) _____

Name (Signature) _____

Z# _____

Part I (8/10)	
1	/20
2	/20
3	/20
4	/20
5	/20
6	/20
7	/20
8	/20
9	/20
10	/20
	/160

Part II (2/3)	
1	/20
2	/20
3	/20
	/40

Net Score	Part I	/160
	Part II	/40
		<hr/>
		/200

Part I: Do any 8 of the following 10 problems.

1. Compute $y(x)$ if

$$(1 + x)y' + y = \cos x$$

$$y(0) = 1$$

2. Solve for $y(x)$

$$y' = 2xy^2 + 3x^2y^2$$

$$y(1) = -1$$

3. Find the general solution for

$$(1 + ye^{xy})dx + (2y + xe^{xy})dy = 0$$

4. Solve for $y(x)$

$$xy' + 6y = 3xy^{4/3}$$

$$y(1) = 1$$

5. Find the general solution for

$$y'' + y' + y = 3 + 2 \cos 2x$$

using the method of **undetermined coefficients**.

6. Find a particular solution for

$$y'' - 4y' + 4y = 2e^{2x}$$

by using **variation of parameters**.

7. Employ the Improved Euler Method with a step size of $h = 0.5$ to estimate $y(1.0)$, where $y(x)$ is the solution of the initial-value problem

$$y' = x - y$$

$$y(0) = 1$$

If you round off any quantity in the computation, keep at least two digits past the decimal point.

8. Solve, using Laplace Transform

$$y'' + 3y' + 2y = t$$

$$y(0) = 0, y'(0) = 2$$

9. Compute the solution for the system

$$\begin{cases} \frac{dx}{dt} = 4x - 3y \\ \frac{dy}{dt} = 6x - 7y \end{cases}$$

subject to $x(0) = 2$, $y(0) = -1$

10. Use Laplace Transforms to solve

$$y'' + 2y' + 10y = f(t)$$

$$y(0) = y'(0) = 0$$

where

$$f(t) = \begin{cases} 2, & 0 \leq t < 3 \\ 0, & t \geq 3 \end{cases}$$

Part II: Do any 2 of the following 3 problems.

1. At time $t = 0$, 200 lbs. of salt is dissolved in a large tank with a capacity of 500 gallons which holds 300 gallons of water. A brine solution is pumped into the tank at the rate of 4 gallons per minute and the well-mixed solution is pumped out at the rate of 2 gallons per minute. If the concentration of the brine solution that is entering the tank is 3 lbs/gallon find an expression for the amount of salt in the tank at time t . How much salt is in the tank when the tank is full?

2. An object of mass 5 kg is dropped from the top of a building 300 m high and encounters air resistance which is proportional to its velocity with constant of proportionality 8 kg/sec. Let t^* be the impact time (the time when the object hits the ground). Find:
- (a) the velocity of the object at time t , $t < t^*$
 - (b) the distance the object falls in time t , $t < t^*$
 - (c) explain (in words) how to find the impact velocity.

3. An automobile starts at point P and moves east until it is 10km



P

from its starting point and is traveling with a velocity of 60 km/hr. The driver then applies the brakes so that the vehicle deaccelerates at a rate proportional to the square of its velocity.

Calculate the distance of the car from point P one minute after the brakes are applied.