

NAME & ZID:

SCORE:

1. (10 points) Verify that for any constant C , the function $y(x) = Ce^{-\cos x}$ is a solution of $\frac{dy}{dx} = y \sin x$.

2. (10 points) Find the solutions to $5x^2y''(x) + 18xy'(x) = 6y(x)$ of the form $y(x) = x^r$ with constant r .

3. (15 points) Solve $3e^{5x} \frac{dy}{dx} = 5(y - 7)^4$.

4. (15 points) Solve $(x^5 e^y + \frac{4}{y}) \frac{dy}{dx} + 5x^4 e^y = \cos x$.

5. (10 points) It is known that the initial-value problem $x^3 \frac{dy}{dx} = \sin^7 x + y^5$, $y(0) = 1$ does not have any solution. Does this contradict with the Basic Existence and Uniqueness Theorem for first-order ODE's? Why?

6. (20 points) Solve $5xz^4 \frac{dz}{dx} = 2x^3 + z^5$.

7. (20 points) A 900-liter tank initially contains 400 liters of brine with 130 kilograms of salt dissolved in it. Brine containing 1 kilogram of salt per liter is pumped into the tank at the rate of 5 liters per second, and the well-mixed brine in the tank flows out at the rate of 3 liters per second. How much salt will the tank contain when it is just full of brine?