1. (10 points) Verify that for any constant $C$, the function $y(x) = Ce^{-\cos x}$ is a solution of $\frac{dy}{dx} = y \sin x$.

2. (10 points) Find the solutions to $5x^2 y''(x) + 18xy'(x) = 6y(x)$ of the form $y(x) = x^r$ with constant $r$. 
3. (15 points) Solve $3e^{5y} \frac{dy}{dx} = 5(y - 7)^4$.

4. (15 points) Solve $(x^5e^y + \frac{4}{y}) \frac{dy}{dx} + 5x^4e^y = \cos x$. 

5. (10 points) It is known that the initial-value problem \( x^3 \frac{dy}{dx} = \sin^7 x + y^5, \ y(0) = 1 \) does not have any solution. Does this contradict with the Basic Existence and Uniqueness Theorem for first-order ODE’s? Why?

6. (20 points) Solve \( 5xz^4 \frac{dz}{dx} = 2x^3 + z^5 \).
7. (20 points) A 900-liter tank initially contains 400 liters of brine with 130 kilograms of salt dissolved in it. Brine containing 1 kilogram of salt per liter is pumped into the tank at the rate of 5 liters per second, and the well-mixed brine in the tank flows out at the rate of 3 liters per second. How much salt will the tank contain when it is just full of brine?