1. (10 points) Use Euler’s Method with step size $h = 1$ to estimate $y(2)$ for the solution $y$ of the IVP $\frac{dy}{dx} = x^3 + y^2$, $y(-1) = 1$.

2. (10 points) Prove that if $y_1(x)$ and $y_2(x)$ are solutions of the ODE $a(x)y''(x) + b(x)y'(x) + c(x)y(x) = 0$ on a non-trivial interval $I$, then for any constants $c_1$ and $c_2$, the function $c_1y_1(x) + c_2y_2(x)$ is also a solution of the ODE on $I$. 
3. (10 points each) Solve the following equations.

\[ 5y^{(5)} - 9y^{(4)} - 2y''' = 48 \]

\[ y^{(4)} - 6y'' + 9y = 0 \]

\[ y''' - 3y'' + y' + 5y = 0 \]
4. (10 points) Give the form of a particular solution to
\[ y^{(4)}(x) + 4y''(x) = x^3 + 5xe^{2x} + x \sin(2x). \]

5. (20 points) Solve \[ y''(x) - 4y'(x) + 4y(x) = \frac{e^{2x}}{x}. \]
6. (20 points) Consider a prolific breed of rabbits with birth rate $\beta = P/200$ and death rate $\delta = P/500$, where $P = P(t)$ is the rabbit population after $t$ months.

i) Show that $P(t) = 1,000P(0)/(1,000 - 3P(0)t)$.

ii) Suppose that $P(0) = 6$. When does doomsday occur (i.e., $P \to +\infty$)?