

NAME & ZID:

SCORE:

1. (10 points) Use the *definition* of Laplace transforms to find $\mathcal{L}\{te^{-6t}\}$.

2. (10 points each) Find the following inverse Laplace transforms.

$$\mathcal{L}^{-1}\left\{\frac{30}{s^3 + s^2 - 6s}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s+1}{s^2 + 6s + 13}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2 + 7)^2}\right\}$$

Hint: use the special convolutions

4. (10 points) Find the Laplace transform $\mathcal{L}\{g(t)\}$ if $g(t) = \begin{cases} 5t & \text{for } 0 \leq t < 3, \\ 7 & \text{for } 3 \leq t < 6, \\ t^2 & \text{for } t \geq 6. \end{cases}$

5. (10 points) A mass weighing 60 slugs is attached to the end of a spring that is stretched 0.04 feet by a force of 500 pounds. An external force in the form $7 \sin(\omega t)$ acts on the mass. Find the frequency of the external force in hertz for which resonance occurs.

6. (20 points) Solve the initial-value problem $y''(t) + 4y(t) = f(t)$, $y(0) = 0$, $y'(0) = 0$, where

$$f(t) = \begin{cases} 0 & \text{if } 0 \leq t < 2, \\ 3 & \text{if } 2 \leq t < 7, \\ 0 & \text{if } t \geq 7. \end{cases}$$

7. (20 points) A mass of 3 slugs is attached to a spring-dashpot system with spring constant 30 pounds per foot and damping constant 21 pounds for each foot per second. At the beginning, the mass is pulled such that the spring is stretched 4 feet and set in motion towards its equilibrium position with a speed of 29 feet per second. Find a formula for the displacement $x(t)$ of the mass and determine when the mass passes its equilibrium position if it does.