

NAME & ZID:

SCORE:

1. (10 points each) Find the following inverse Laplace transforms.

$$\mathcal{L}^{-1}\left\{\frac{s+5}{s^3+3s^2-4s}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s-3}{s^2+4s+11}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s^2+5)^2}\right\}$$

Hint: use special convolutions

2. (10 points) Compute the convolution $t * e^{3t}$ using the *definition*.

3. (10 points) Find the Laplace transform $\mathcal{L}\{g(t)\}$ if $g(t) = \begin{cases} t^2 & \text{for } 0 \leq t < 3, \\ 8 & \text{for } 3 \leq t < 7, \\ 5t & \text{for } t \geq 7. \end{cases}$

4. (10 points) A mass weighing 60 slugs is attached to the end of a spring that is stretched 0.05 feet by a force of 507 pounds. An external force in the form $8 \cos(\omega t)$ acts on the mass. Find the frequency of the external force in hertz for which resonance occurs.

5. (20 points) Solve the initial-value problem $y''(t) + 5y(t) = f(t)$, $y(0) = 0$, $y'(0) = 0$, where

$$f(t) = \begin{cases} 0 & \text{if } 0 \leq t < 3, \\ 7 & \text{if } 3 \leq t < 6, \\ 0 & \text{if } t \geq 6. \end{cases}$$

6. (20 points) A mass of 4 slugs is attached to a spring-dashpot system with spring constant 49 pounds per foot and damping constant 28 pounds for each foot per second. At the beginning, the mass is pulled such that the spring is stretched 6 feet and set in motion towards its equilibrium position with a speed of 26 feet per second. Find a formula for the displacement $x(t)$ of the mass and determine when the mass passes its equilibrium position if it does.