1. Let A be a subset of a topological space such that each $x \in A$ has a neighborhood U_x satisfying $U_x \subseteq A$. Prove that A is open.

2. Let X be a set. Prove that the collection

$$\mathcal{T}_{cc} = \{U \subseteq X;\ X - U \text{ is countable or the whole } X\}$$

is a topology on X. Is the collection

$$\mathcal{T}_{\infty} = \{U \subseteq X;\ X - U \text{ is infinite or empty or the whole } X\}$$

also a topology on X?

3. Prove that the collection

$$\mathcal{T} = \{\emptyset, \mathbb{R}\} \cup \{(-\infty, b), (-\infty, b] :\ b \in \mathbb{R}\}$$

is a topology on $\mathbb{R}$.