

1. Show that the countable collection

$$\mathcal{B}_7 = \{(a, b) : a < b, a, b \in \mathbb{Q}\}$$

is a basis for the standard topology \mathcal{T}_s on \mathbb{R} .

2. Show that the countable collection

$$\mathcal{B}_8 = \{[a, b) : a < b, a, b \in \mathbb{Q}\}$$

is a basis for a topology on \mathbb{R} , to be called the **rational lower limit topology** on \mathbb{R} and denoted by \mathcal{T}_{rl} . Prove that \mathcal{T}_{rl} is strictly stronger than \mathcal{T}_s , but strictly weaker than \mathcal{T}_l . Also, describe the elements of \mathcal{T}_{rl} .