1. Prove that the collection

$$\mathcal{S} = \bigcup_{\alpha \in J} \Big\{ \prod_{\beta \in J} U_\beta : \; U_\alpha \subseteq X_\alpha \text{ open}, \, U_\beta = X_\beta \text{ if } \beta \neq \alpha \Big\}$$

is a subbasis for the product topology on $\prod_{\alpha \in J} X_\alpha$.

2. Prove that if $\mathcal{S}_\alpha$ is subbasis for the topology on X_α for each $\alpha \in J$, then the collection

$$\mathcal{R} = \Big\{ \prod_{\alpha \in J} S_\alpha : \; S_\alpha \in \mathcal{S}_\alpha \, \forall \, \alpha \in J \Big\}$$

is a subbasis for the box topology on $\prod_{\alpha \in J} X_\alpha$, and the collection

$$\mathcal{Q} = \bigcup_{\alpha \in J} \Big\{ \prod_{\beta \in J} S_\beta : \; S_\alpha \in \mathcal{S}_\alpha, \, S_\beta = X_\beta \text{ if } \beta \neq \alpha \Big\}$$

is a subbasis for the product topology on $\prod_{\alpha \in J} X_\alpha$.