

1. Prove that if  $\mathcal{B}$  and  $\mathcal{C}$  are bases for the topologies on  $X$  and  $Y$ , respectively, then a map  $f : D \subseteq X \rightarrow Y$  is continuous at  $x_0 \in D$  if and only if for each  $C \in \mathcal{C}$  containing  $f(x_0)$ , there exists a  $B \in \mathcal{B}$  containing  $x_0$  such that  $f(B \cap D) \subseteq C$ .
2. Let  $\mathbb{R}_l$  be the topological space consisting of  $\mathbb{R}$  and the lower limit topology on it. Describe the continuous maps  $f : \mathbb{R} \rightarrow \mathbb{R}_l$  and the continuous maps  $f : \mathbb{R}_l \rightarrow \mathbb{R}_l$ .
3. Prove that if  $\mathcal{C}$  is a basis for the topology on  $Y$ , then a map  $f : X \rightarrow Y$  is continuous if and only if  $f^{-1}(C)$  is open in  $X$  for each  $C \in \mathcal{C}$ ; if  $\mathcal{R}$  is a sub-basis for the topology on  $Y$ , then  $f$  is continuous if and only if  $f^{-1}(R)$  is open in  $X$  for every  $R \in \mathcal{R}$ .