1. Let X be a set. Show that

$$d(x,y) = \begin{cases} 0 & \text{if } x = y, \\ 1 & \text{if } x \neq y \end{cases}$$

defines a metric on X, to be called the **discrete metric** on X.

2. Consider $f(r) = r/(1+r)$ for $r > -1$.
   (1) Directly verify that $f(a+b) \leq f(a) + f(b)$ for $a, b > -1$ satisfying $ab \geq 0$.
   (2) Use the Mean Value Theorem to show that $f(a+b) - f(a) \leq f(b)$ for $a \geq b \geq 0$.

3. Show that if (X, d) is a metric space, then

$$\tilde{d}(x,y) = \frac{d(x,y)}{1 + d(x,y)}$$

also defines a metric on X. *Hint*: Use 2.