Math 211  Final Exam  FALL 2009

Name (print): __________________________
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Section: ________

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TOTAL / 200
1. [20 pts.] Find the derivative of each of the following functions. (Do not simplify).

(a) \( f(x) = \left( \frac{2}{x^2} + 5\sqrt{x} + e^{5x} \right)^2 \)

(b) \( f(x) = \ln \left( \frac{1 - x^2}{1 + 2x^2} \right)^5 \)

(c) \( f(x) = 5e^{\sqrt{x}-2} \)

(d) \( f(x) = \frac{x^3 + 3x}{3x^2 - 4} \)

(e) \( f(x) = x^2 \ln(2x^2 + 5) \)
2. [10 pts.]

(a) Let the function \( f(x) \) be defined by

\[
f(x) = \begin{cases} 
  \frac{x^2 - 4}{x - 2} & x \neq 2 \\
  4 & x = 2 
\end{cases}
\]

Is this function \( f(x) \) continuous at \( x = 2 \)? Justify your answers.

(b) [5 pts.] \( \lim_{x \to 1} (x^2 + 2x)^5 \)

(c) [5 pts.] \( \lim_{x \to \infty} \frac{x^2 - 3x + 2}{-2x^2 + 5} \)
3. [15 pts.] Let $f(x) = x^4 - 2x^2 + 1$. Find absolute maximum and minimum values of $f(x)$ on the interval $[-2, 2]$.

4. [15 pts.] Suppose 1000gm of Strontium-90 becomes 500gm after 10 days. After how many days will 1000gm be 30gms?
5. **[20 pts.]** Given \( f(x) = x^3 - 3x^2 + 1 \)

   (i) Find all the critical points.

   (ii) Find the relative maxima/minima of the function using a derivative test.

   (iii) Use the 2nd derivative test to determine where the graph of \( f(x) \) is concave upward and concave downward. Find all the inflection point(s).
(iv) Draw a rough sketch of the graph of \( f(x) \) using the information in (i)-(iii).

6. [15 pts.] Suppose \$1000.00\) is invested in a savings account in which interest is compounded continuously at 7.2% per year.

(a) What is the balance after 3 years?

(b) When will \$1000.00\) be \$2000.00?\)
7. [20 pts.] A small furniture manufacturer estimates that the cost (in dollars) of producing $x$ units of a particular chair is given by

$$C(x) = 10,000 + 4x + 0.01x^2$$

and the revenue for the production of $x$ chairs is

$$R(x) = 48x - 0.012x^2.$$ 

(a) Find the profit function $P(x)$.

(b) Find the number of chairs, that will maximize the profit.
8. [20 pts.] Evaluate

(a) \[ \int_{1}^{2} \left( 3x^2 - \frac{1}{x^2} + 2 \right) \, dx \]

(b) \[ \int_{1}^{2} \left( \frac{2}{x} - 5e^x \right) \, dx \]
9. [20 pts.] Compute the following integrals:

(a) \( \int \frac{(x + 1)}{(x^2 + 2x)^{10}} \, dx \)

(b) \( \int 2xe^{x^2+2} \, dx \)

(c) \( \int \left( x^3 + 3x - \frac{2}{x} \right) \, dx \)

(d) \( \int \left( \frac{\sqrt{x}}{x^2} + \frac{3}{x^2} - 4x^3 + 2 \right) \, dx \)
10. [20 pts.] Find the area of the region bounded by the curves \( y = 2x^2 - 2x + 1 \) and \( y = x^2 + 1 \).

11. [15 pts.] Find \( f(x) \) where \( f'(x) = 4x^3 + 2x + e^x \) and \( f(0) = 2 \).