1. Assume the following constraints are satisfied:
\[
\begin{align*}
2x_1 - x_2 & \geq 2 \\
x_1 + x_2 & \leq 6 \\
x_1 + 2x_2 & = 8 \\
x_1, x_2 & \geq 0
\end{align*}
\]
(a) Use the graphical method to determine the maximizer and the minimizer of 
\[f(x_1, x_2) = 3x_1 - 4x_2.\]
(b) Using the same constraints, convert the constraints to the form of a primal problem.

2. Use the simplex algorithm to solve the problem:
\[
\begin{align*}
\text{max } x_1 - 2x_2 + 3x_3 \; \text{ when } \\
& \quad x_1 + x_3 \leq 4 \\
& \quad x_2 + x_3 \leq 5 \\
& \quad x_1, x_2, x_3 \geq 0
\end{align*}
\]
Make sure that each step employed is the step suggested by the simplex algorithm.

3. Write down but do not solve the dual problem of the following primal problem:
\[
\begin{align*}
\text{max } x_1 - x_3 + 2x_4 \; \text{ when } \\
& \quad 2x_2 - x_4 \leq 12 \\
& \quad 3x_1 - x_2 + 4x_3 \leq 20 \\
& \quad x_2 - x_3 + x_4 + x_5 \leq 25 \\
& \quad x_i \geq 0, i \in \{1, \ldots, 5\}
\end{align*}
\]

4. Complete the first phase of the two-phase method to find a feasible point of the primal problem with constraints:
\[
\begin{align*}
x_1 - 2x_2 + 3x_3 & \leq -6 \\
2x_1 + x_2 & \leq 12 \\
x_1, x_2, x_3 & \geq 0
\end{align*}
\]
Make sure that each step employed is the step suggested by the first phase of the two-phase method.
5. Write down the primal problem corresponding to the following dual problem. Apply the simplex method to the primal problem. From the final tableau, find the minimizer

\[ \lambda^T = [\hat{\lambda}_1 \quad \hat{\lambda}_2 \quad \hat{\lambda}_3] : \]

\[
\begin{align*}
\min 4\lambda_1 + 3\lambda_2 + 2\lambda_3 \quad \text{when} \\
2\lambda_1 + \lambda_2 & \geq 2 \\
\lambda_3 & \geq 1 \\
\lambda_1 + \lambda_3 & \geq 6 \\
\lambda_1, \lambda_2, \lambda_3 & \geq 0
\end{align*}
\]

6. The problem:

\[
\begin{align*}
\max 25x_1 + 30x_2 + 18x_3 \quad \text{when} \\
2x_1 + 3x_2 + 4x_3 & \leq 60 \\
3x_1 + x_2 + 5x_3 & \leq 45 \\
x_1 + 2x_2 + x_3 & \leq 50 \\
x_1, x_2, x_3 & \geq 0
\end{align*}
\]

has a maximizer \( \hat{x}_1 = \frac{75}{7}, \hat{x}_2 = \frac{90}{7}, \hat{x}_3 = 0 \). Use nothing but complementary slackness to find the dual solution \( \hat{\lambda} \).