1. (20 pts.) Evaluate the following derivatives. Don’t simplify your answers.

(a) \( f(x) = \frac{1}{(x^3 + 3x^2 + 2)^2} = \left(\frac{x^3 + 3x^2 + 2}{x^3 + 3x^2 + 2}\right)^{-2} \)

\[ f'(x) = -2 \left(\frac{x^3 + 3x^2 + 2}{x^3 + 3x^2 + 2}\right)^{-3} \cdot (3x^2 + 6x) \]

(b) \( f(x) = \ln \left\{ \frac{(x^3 + 3x)(5x^2 - 2)}{(3x + 2)} \right\} = \ln \left(\frac{x^3 + 3x}{3x + 2}\right) + \ln \left(\frac{5x^2 - 1}{x^3 - 1}\right) - \ln \left(\frac{3x + 2}{x^3 - 1}\right) \)

\[ f'(x) = \frac{3x^2 + 6x}{x^3 + 3x^2 + 2} + \frac{10x}{5x^2 - 1} - \frac{3}{3x + 2} \]

(c) \( f(x) = \ln \left( e^{tx^2} \sqrt{x^2 + 1} \right) = \ln \left( e^{tx^2} \right) + \ln \left(\sqrt{x^2 + 1}\right) = \frac{4}{2} + \frac{1}{2} \ln \left(\frac{x^2 + 1}{x^2 + 1}\right) \)

\[ f'(x) = \frac{4}{2} x + \frac{1}{2} \cdot \frac{2x}{x^2 + 1} \]

(d) \( f(x) = e^{(x^2 + 2x)} \cdot (5x - 2x^2 + 3) \)

\[ f'(x) = e^{(x^2 + 2x)} \cdot (5 - 4x) + (2x + 3) e^{(x^2 + 2x)} \cdot (5x - 2x^2 + 3) \]

\( \text{(Product Rule)} \)
2. (15 pts.) Find the equation of the tangent line to the curve \( y = (x - 1) \ln(x + 2) \) at the point \( x = -1 \).

\[
\frac{dy}{dx} = (x-1) \frac{1}{x+2} + \ln(x+2).
\]

At \( x = -1 \),
\[
\left. \frac{dy}{dx} \right|_{x=-1} = -(2) \left( \frac{1}{-1} \right) + \ln(1) = -2 + 0 = -2
\]

When \( x = -1 \),
\[
y = -(2) \ln(-1+2) = -2 \ln(1) = 0
\]

The tangent line therefore has slope \(-2\) and passes through \((-1, 0)\).

Equation: \[(y-0) = -2(x-(-1))\]

OR \[y = -2(x+1)\]

3. (15 pts.) The population of a colony of bacteria after \( t \) hours is given by \( P(t) = 100e^{.7t} \).

(a) Find the rate of growth of the population after 10 hours.

Since \( \dot{p} = .7 \), the growth rate is 70 \( \% \) per hour (constant).

The rate of growth \( \dot{P}(t) \) is 
\[
\dot{P}(t) = 100(.7)e^{.7t} = 70e^{.7t}
\]

And \( P(10) = 70e^{(10)(.7)} = 70e^{7} \)

(b) At what time will the population double?

\[
\Rightarrow P(t) = 200 \Rightarrow 200 = 100e^{.7t} \Rightarrow e^{.7t} = \frac{200}{100} = 2 \Rightarrow (.7)t = \ln2
\]

so population doubles in \( t = \frac{\ln2}{.7} \) days.

(c) How large will the population of the colony of bacteria be after 10 hrs.?

\[
P(10) = 100e^{(10)(.7)} = 100e^7
\]

\[\checkmark \text{In part (A), the growth rate is 70\% per hour. But the problem should have asked for the rate of change.}\]
4. (15 pts.) A construction company has 840 ft. of chain-link fence that is used to enclose storage areas for materials at construction sites. The supervisor wants to setup two identical rectangular storage areas sharing a common fence (see the figure).

Assuming that all fencing is used:

(a) Express total area \( A(x) \) enclosed by both pens as a function of \( x \).

\[
\begin{align*}
\text{Constraint: Amount of fence} & = 840 \\
3x + 4y & = 840 \\
y & = \frac{840 - 3x}{4} \\
A(x) & = x \cdot \left( y \cdot \frac{1}{2} \right) \\
& = x \cdot \frac{1}{2} \left( 840 - \frac{3}{4}x \right) \\
& = 420x - \frac{3}{4}x^2
\end{align*}
\]

(b) Find the value of \( x \) that will maximize \( A(x) \) (area).

\[
\begin{align*}
A'(x) & = 420 - 3x \\
A'(x) = 0 & \Rightarrow 420 = 3x \Rightarrow x = \frac{420}{3} = 140 \text{ feet}
\end{align*}
\]

(c) What is the largest total area that can be enclosed.

\[
A(140) = 420 \cdot 140 - \frac{3}{2} (140)^2
\]

\[
= 140 \left( 420 - \frac{3}{2} \cdot 140 \right) = 140 \left( 420 - 3 \cdot 70 \right)
\]

\[
= 140 \left( 420 - 210 \right) = (140) \cdot (210) \text{ square feet}
\]
5. (15 pts.) 20 gms of a certain radioactive material decays to 10 gms in 10 days. After how many days will just 6 gms remain?

\[ \ln(5) = -0.693; \ln(0.3) = -1.204 \]

\[ \rho(t) = 20 e^{-kt}, \quad \rho(10) = 10 \Rightarrow \frac{10}{20} = e^{-10k} \]

\[ \Rightarrow -10k = \ln(0.5) \Rightarrow k = \frac{\ln(0.5)}{-10} = \frac{\ln(2)}{10} \]

\[ \rho(t) = 6 \Rightarrow 6 = 20 e^{\frac{-ln(2)}{10}t} \Rightarrow \frac{6}{20} = e^{\frac{-ln(2)}{10}t} \Rightarrow t = \frac{10 \ln(6/5)}{-ln(2)} \text{ days} \]

6. (10 pts.) Compute the following integrals.

(a) \[ \int \left( 2x^3 + 5x^2 + \frac{2}{x} + e^{-5x} \right) \, dx = 2 \frac{x^4}{4} + 5 \frac{x^3}{3} + 2 \ln x + \left( \frac{-1}{5} \right) e^{-5x} + C \]

(b) \[ \int \left( \frac{2x^4 + 3x^3 + 5}{x^4} \right) \, dx = \int \left( 2 + \frac{3}{x} + 5x^{-4} \right) \, dx \]

\[ = 2x + 3 \ln x + 5 \frac{x^{-3}}{-3} + C \]

7. (10 pts.) Find \( f(x) \) where \( f'(x) = 3x^2 + 5x + 2 \) and \( f(0) = 1 \).

\[ f(x) = \begin{cases} (3x^2 + 5x + 2) \, dx = 3 \frac{x^3}{3} + 5 \frac{x^2}{2} + 2x + C = x^3 + \frac{5}{2} x^2 + 2x + C \quad \text{for some } C \end{cases} \]

Now find \( C \) such that \( f(0) = 1 \):

\[ f(0) = 0^3 + \frac{5}{2} 0^2 + 2(0) + C = C \Rightarrow C = 1 \]

so \( C = 1 \) and \( f(x) = x^3 + \frac{5}{2} x^2 + 2x + 1 \)