MATH 211  
FINAL EXAM  
5/3/97  
NO CALCULATORS!

1. (30 points) Find the derivative of each of these functions. You do not need to simplify your answers.

(a) \( f(x) = x^2 + \frac{1}{x^2} \)
(b) \( f(x) = (x^5 + \ln x)^4 \)
(c) \( f(x) = \frac{x^3 - 1}{x^2 + 1} \)
(d) \( f(x) = e^{x^2} \sqrt{x^3 + 5} \)

2. (15 pts) Find the following limits.

(a) \( \lim_{x \to 6} \frac{x^2 - 4x - 12}{x - 6} = \)
(b) \( \lim_{x \to 3} \frac{x^2 - 4x - 12}{x^2 - 8x + 12} = \)

3. (20 pts) Let \( f(x) = \frac{1}{2}x + \frac{2}{x} \).

(a) Find and classify the extreme points (relative maximum and minimum points) of \( y = f(x) \).
(b) Graph the given function \( f(x) \), using your knowledge of calculus.

4. (30 pts) Find the following integrals.

(a) \( \int_1^4 \frac{1}{x^2} \, dx = \)  
(b) \( \int_0^5 e^{-2t} \, dt = \)  
(c) \( \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \, dx = \)

5. (15 pts) Find the area bounded by the curves \( y = 2x^2 + x - 5 \) and \( y = x + 3 \) and the vertical lines \( x = -2 \) and \( x = 2 \). (A sketch of the curves may help you.)

6. (15 pts) A store manager wants to build a 600-square-foot rectangular enclosure. Three sides of the enclosure will be built of redwood fencing, at a cost of $14 per foot. The fourth side will be built of cement blocks, at a cost of $28 per foot. Find the length and width of the enclosure that will minimize the total cost of the building materials.

7. (15 pts) The demand equation for a certain product is \( p = 200 - 3x \), where \( p \) is the price and \( x \) is the number of units produced. The cost function is \( C(x) = 75 + 80x - x^2 \), where \( 0 \leq x \leq 40 \).

(a) Determine the level of production that will maximize the profit.
(b) Suppose that the government imposes a tax of $4 per unit produced. Find the new price that maximizes the profit.

8. (15 pts) Five grams of a certain radioactive material decays to 3 grams in 1 year. After how many years will just 1 gram remain?

9. (7 pts) Let \( P(t) = P_0 e^{kt} \). Compute \( P'(t) \) and show that \( P'(t) = kP(t) \).

10. (8 pts) Find the equation of the tangent line to the curve \( y = x \ln x \) at \( x = 1 \).

11. (20 pts) Find the derivative of each of the following functions.

(a) \( f(x) = \sqrt{x + \sqrt{x}} \)
(b) \( f(x) = \ln (\sqrt{x^3} + 3) \)

12. (10 pts) Using the limit definition of the derivative (not the power formula), find \( f'(x) \) if \( f(x) = x^3 \).