1. (1.1, p21, #7) Let $S$ be a nonempty finite set with a binary operation $*$ that satisfies the associative law. Show that $S$ is a group if $a * b = a * c$ implies $b = c$ and $a * c = b * c$ implies $a = b$ for all $a, b, c \in S$. What can you say if $S$ is infinite?

2. (1.1, p21, #8) Prove that if $G$ is a group and $a, b \in G$, then the equations $ax = b$ and $xa = b$ have unique solutions. Conversely, prove that if $G$ is a nonempty set with an associative binary operation in which the equations $ax = b$ and $xa = b$ have solutions for all $a, b \in G$, then $G$ is a group.

3. (1.1, p21, #8) Let $G$ be a set with an associative binary operation $\ast$. Prove that $G$ is a group if (i) there exists a left identity $e \in G$ such that $e \ast a = a$ for each $a \in G$, and (ii) for each $a \in G$ there exists a left inverse $b \in G$ such that $b \ast a = e$.

4. (1.1, p21, #20) Let $F$ be a field with $q$ elements. Find $|GL_n(F)|$.

5. (1.2, p31, #13) Show that in $SL_2(\mathbb{Z}_3)$ the elements \[
\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}
\] generate a subgroup isomorphic to the quaternion group $Q_8$.

6. (1.2, p31, #20) Let $G$ be a group with $p^k$ elements, where $p$ is a prime number and $k$ is a positive integer. Prove that $G$ has a subgroup of order $p$. 