1. Let $A$, $B$, and $C$ be subsets of the universal set $U$. Draw Venn diagrams illustrating the following:

First, we have a square representing the universe $U$ and three circles, one for each of $A$, $B$ and $C$. Each of pair of circles intersects in an “orange slice”, and all three circles (and all 3 orange slices) intersect in a triangle.

(a) $(A \cap B) - C$
Region is the orange slice created by the intersection of $A$ and $B$ excluding the “triangle”.

(b) $(A - B) \cup C$
$A-B$ is all of $A$ excluding the orange slice $A \cap B$. When we union this with $C$, we get all of $C$ plus the rest of the part of $A$ that is outside of $B$.

2. Let $A = \{1, 2, 3\}$ and $B = \{a, b\}$. How many elements are in $A \times B$? List them.

There are 3 elements in $A$ and 2 in $B$ - thus there are 3 choices for a first coordinate and 2 for a second, for a total of $3 \times 2 = 6$ choices or pairs. They are $(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)$.

3. Let $B = \{1, 2, 3\}$, and $C = \{\{1\}, 2, 3\}$. Find $A \cap B$. For each of the following determine whether it is an element, subset or neither of $A$ and of $B$. 1, $\{1\}$, 2, $\{3\}$.

$A \cap B = \{2, 3\}$.
1 is an element of $A$. It is neither an element or a subset of $B$.
$\{1\}$ is a subset of $A$, and an element of $B$.
$\{3\}$ is a subset of both sets.

4. Let $U = \{1, 2, 3, 4, 5, 6\}$, and let $A = \{1, 2, 3\}, B = \{2, 3, 4\}, C = \{4, 5, 6\}$. Find

(a) $A \cup \bar{B}$
NOTE THE “bar” over $B$. Answer is $\{1\}$.

(b) $A - B$
Answer is $\{1\}$.

(c) The complement of $(A \cup C)$
It’s the empty set, $\emptyset$.

5. Give truth tables for $p \lor q$, $p \land q$ and $p \Rightarrow q$.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \lor q$</th>
<th>$p \land q$</th>
<th>$p \Rightarrow q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>
6. State whether each of the following is True or False (using basic facts that you know).

(a) This is the month of May, or it is winter. **True**
(b) If it is 1845, then everyone in MA 206 will get an on the final exam. **T**

7. Give useful negations, in words, of each of the following.

(a) If Rommel fights Montgomery, then Montgomery will win.
    **Rommel fights Mongomery and Montgomery does not win.**
(b) Joe has packed his bags, and he is ready to go.
    **Joe has not packed his bags or he is not ready to go.**

8. Evaluate the following: $P(n, 0)$, $P(n, 1)$, $P(6, 3)$.

$$P(n, 0) = \frac{n!}{(n-0)!} = \frac{n!}{n!} = 1, \quad P(n, 1) = \frac{n!}{(n-1)!} = n, \quad P(6, 3) = \frac{6!}{(6-3)!} = \frac{6!}{3!} = 6 \cdot 5 \cdot 4 = 120$$

9. An I. D. “number” at Buck’s Bank consists of two letters followed by 3 digits. If the I. D. number cannot have an ‘O’ (Oh) in it, and cannot end with an odd digit, then how many possible I. D. Numbers are there?

   Use a box model with five boxes to be filled in.
   
   There are 25 choices for a letter in the first box - anything but O (oh).
   Same for the second box.
   Third and fourth boxes can have one of ten digits 0-9.
   Fifth box can only use one of five choices - 2, 4, 6, or 8.
   So there are total of $25 \cdot 25 \cdot 10 \cdot 10 \cdot 5$ choice of ID numbers.

10. A production committee at the Iron Works consists of one member of management, two senior workers, and two junior workers. If there are seven managers, 10 senior workers, and six junior workers eligible to serve on the committee, then in how many ways can the committee be formed?

   Notice the trick to this. You saw it several times in class.
   First there are 7 ways to choose a manager (no trick here).
   If we use a box model to choose the two senior workers, then there are 10 choices for the first manager chosen, and 9 for the second. But note, HERE’S the TRICK, each choice of two has been counted twice - once as a first choice and once as a second choice (we called choices where order wasn’t important “combinations”). So there are really $(10 \cdot 9)/2 = 45$ choices.
   The answer is 7 for a manager, 45 for the two seniors, and 15 choices for the two juniors, for a total of $7 \cdot 45 \cdot 15$ ways to pick the committee.

11. There are six fourth graders and five third graders on a field trip. In how many ways can they line up to go into a museum, if the classes are to remain together?
First note that there two choices you might overlook. There are two choices as to which class is first in line, 4th or 5th graders. That’s a TRICK similar to that in the last problem.

Now, the ordering of the separate classes is just a matter of counting the permutations (ways of lining them up). There are 6! ways of ordering the 4th graders and 5! ways of ordering the 5th graders.
For a total of $2 \cdot 6! \cdot 5!$

12. A palindrome is a number which reads the same front to back as it does back to front.

(a) How many 3 digit palindromes are there?

Use a box model. First and last digit must be the same, so there are 10 choices for the (same) 1st and last digit, and 10 choices for the middle. Total is 100.

(b) How many 4 digit palindromes?

Similar to last. 10 ways of choosing the 1st and last, 10 ways to choose the 2nd and 3rd, for a total of 100 palindromes.

13. Let $\mathcal{R}$ be a relation on the set $S$. Define the following:

(a) $\mathcal{R}$ is reflexive.

$\mathcal{R}$ is reflexive if for all $r \in S$, $r \mathcal{R} r$ (or $(r, r) \in \mathcal{R}$).

(b) $\mathcal{R}$ is transitive.

$\mathcal{R}$ is transitive of whenever there are $r, s, t \in S$ such that $r \mathcal{R} s$ and $s \mathcal{R} t$, then $r \mathcal{R} t$ also. (or $(r, s), (s, t) \in \mathcal{R}$ implies $(r, t) \in \mathcal{R}$.)

(c) $\mathcal{R}$ is an equivalence relation (list properties).

An equivalence relation is Reflexive, Symmetric and Transitive. (What’s "symmetric"?)

14. How does a partial order differ from an equivalence relation? (That is, what property is different and how is it different.) Give an example of a partial order that is not a total order.

Both are reflexive and transitive. Equivalence relations are symmetric. Partial orders are antisymmetric, that is, if $x \leq y$ and $y \leq x$ then $x = y$.

The set of all subsets of a given set under the relation $\subseteq$ is a partial order, which is not a total order if the set has more than one element.

15. Give examples of functions $f, g, h, k$, such that

For each function we will denote the domain by $D$ and codomain by $C$. We will use pairs for the functional notation.

We will then give a second example which uses the real numbers.
(a) \( f \) is 1-1 but not onto.

\[
D = \{1\}, \quad C = \{1, 2\}. \quad f = \{(1, 1)\}.
\]

Let \( D \) be the non negative real numbers and let \( C \) be all reals.
Then the inclusion map \( f : D \to C, \ f(x) = x \) is one-to-one, but not onto - doesn’t send anything to the negatives.

(b) \( g \) is onto, but not 1-1.

\[
D = \{1, 2\}, \quad C = \{1\}. \quad f = \{(1, 1), (2, 1)\}.
\]

Let \( D \) be the set of all integers, and \( C \) the set of all non-negative integers. Then \( f(x) = |x| \) (absolute value) is onto, but not 1 to 1.
How would you justify this?

\( h \) is not 1-1 and \( h \) is not onto. \( D = \{1, 2\}, \quad C = \{1, 2\}. \quad f = \{(1, 1), (2, 1)\}.
\]

Let \( D \) be the set of all integers, and \( C \) the set of all negative integers. Then \( f(x) = |x| \) (absolute value) is not onto, but not 1 to 1. How would you justify this?

(c) \( k \) is 1-1 and onto.

How about the identity function from a set to itself?

Be specific as to range and domain.

16. If \( f = \{(1, x), (2, y), (3, x)\} \) and \( g = \{(x, \alpha), (y, \beta)\} \), then find the composite function \( g \circ f \).

\[
g \circ f = \{(1, \alpha), (2, \beta), (3, \alpha)\}
\]

17. Use the Euclidean algorithm to find the G.C.D of 168 and 34. Then find \( x \) and \( y \) such that \( 168x + 34y = gcd(168, 34) \).

\[
168 = 4 \times 34 + 32 \\
34 = 1 \times 32 + 2 \\
32 = 16 \times 2
\]

The gcd is 2. We now work backwards.

18. give two equivalent definitions of \( a \) is congruent to \( b \) modulo \( n \). How is this denoted? Which of the following are congruent to \( 5 \), mod 7 : 2, 12, -1, -1, -16, 145?

Any of the following are equivalent

\( a \) is congruent to \( b \) modulo \( n \) if \( n - (b-a) \);

\( a \) is congruent to \( b \) modulo \( n \) if \( a \) and \( b \) have the same remainder when divided by \( n \);

\( a \) is congruent to \( b \) modulo \( n \) if \( a = b + nk \), for some integer \( k \).

12, -16, 145 are the only one congruent to 5 mod 7.

It is denoted by \( a \equiv b \) (mod \( n \)) or \( a \equiv_n b \)
19. The relation of congruence mod 5, \( \equiv_5 \) is restricted to the subset \( A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} \). The equivalence classes of \( \equiv_5 \) form a partition of the set \( A \), find that partition.

\[
\{\{0, 5, 10\}, \{1, 6, 11\}, \{2, 7, 12\}, \{3, 8\}, \{4, 9\}\}
\]

20. Define a Hamiltonian circuit of a graph.

A Hamiltonian circuit is a path which starts and ends at the same vertex and passes through every other vertex exactly once.

21. When does a multigraph have an Eulerian circuit? An Eulerian path? Find a circuit or path (or explain why not) 19 p177

The multigraph has an Eulerian circuit if every vertex has even degree. It has an Eulerian path if there are exactly 2 vertices of odd degree. You do the problems.

22. Find an Eulerian circuit using the algorithm for Fig 4.34 on p 186. You do it.

23. Find the minimal distance from \( K \) to any other vertex for the graph in Prob 4, p 190. Should be familiar.


25. Find an Eulerian circuit using the algorithm for Fig 4.34 on p 186.

26. Find the minimal distance from \( K \) to any other vertex for the graph in Prob 4, p 190. Then find a spanning tree, using your work. Now do a depth first search and enumerate the vertices. When you have to make a decision, go in alphabetical order.

27. Explain the differences between a graph and a multigraph, give an example of a multigraph that is not a graph.

A graph has at most one edge between any two vertices. A multigraph can have more than one edge between a pair of vertices. Your example should have multiple edges between a couple of vertices, and/or loops at vertices.

28. Give two conditions that insure that a connected graph is a tree.

Several ways. The graph is connected and acyclic (know what that means), or the graph is connected and there are no simple circuits, there is a a unique simple path between any two vertices.
29. draw a tree with 6 vertices, where one vertex has degree 3 and all others have lesser degree.
   Should be obvious if you’ve got it.

30. Which of the following is not a statement?
   (a) Smoking may be hazardous to your health. Yes
   (b) Don’t slam the door. No
   (c) Some dogs are nice. Yes (if “nice” has an agreed on meaning.)
   (d) Mathematics is a required subject. Yes
   (e) Automobiles can rust. Yes

31. What is the negation of the following statement?
   Some students work hard or some professors are not entertaining.

   All students do not work hard And all professors are entertaining.

32. What is the inverse of
   If it is Friday, then I am taking a test

   If it is not Friday then I am not taking a test.

   What is the negation?

   It is Friday and I am not taking a test.

33. Which of the following is a conjunction? (d) only
   (a) The sun shines bright on my old Kentucky Home.
   (b) Dogs bark at midnight or the moon is not full.
   (c) If you are physically fit, then you can run a mile.
   (d) Donny’s stereo is too loud and he has bad manners.
   (e) I don’t know how you can eat that stuff.