7.1
Let $H = I - \frac{2uu^T}{u^Tu}$ be a Householder matrix. Then prove the followings
(i) $Hu = -u$.
(ii) $Hv = v$ if $u^Tu = 0$.

7.2
Let $x$ be an $n$-vector. Develop an algorithm to compute a Householder matrix $H = I - \frac{2uu^T}{u^Tu}$ such that $Hx$ has zeros in positions $(r + 1)$ through $n$; $r < n$.
How many flops will be required to implement this algorithm?
Given $x = (1, 2, 3)^T$, apply your algorithm to construct $H$ such that $Hx$ has a zero in the third position.

7.3
(a) Develop algorithms for implicitly computing (i) $HA$, (ii) $AH$, and explicitly computing (iv) $Q = H_1H_2\ldots H_r$, where the matrices $H$ and $H_i$ for $i = 1, 2, 3, \ldots, r$ are Householder matrices of order $n$, and $A$ is an arbitrary rectangular matrix of appropriate sizes.

7.4
(a) Given the Householder vector $u = (1, 1, 1)^T$ and
$$A = \begin{pmatrix}
1 & 2 \\
3 & 4 \\
5 & 6
\end{pmatrix},$$
compute $HA$ and $A^TH$ using the algorithms developed in Exercise 7.3 (a).

7.6
(a) Show that it requires $2n^2 \left(m - \frac{n}{3}\right)$ flops to compute $R$ in the QR factorization of an $m \times n$ matrix $A$ $(m \geq n)$ using Householder’s method, and that if $Q$ is needed, then the count is $4 \left(m^2n - mn^2 + \frac{n^3}{3}\right)$ flops.

7.21
Let $U$ have orthogonal columns. Then using SVD, prove that
(i) $\|AU\|_2 = \|A\|_2$,
(ii) $\|AU\|_F = \|A\|_F$.

7.22
Let $U\Sigma V^T$ be the SVD of $A$. Then prove that $\|U^TAV\|_F^2 = \sum_{i=1}^p \sigma_i^2$, where $\sigma_i$ are the singular values of $A$.
Let $A$ be an $m \times n$ matrix.

(a) Using the SVD of $A$, prove that

(ii) $\text{Cond}_2(A^T A) = (\text{Cond}_2(A))^2$;

(iii) $\text{Cond}_2(A) = \text{Cond}_2(U^T A V)$, where $U$ and $V$ are orthogonal.

(b) Let $\text{rank}(A_{m \times n}) = n$, and let $B_{m \times r}$ be a matrix obtained by deleting $(n - r)$ columns from $A$. Then prove that $\text{Cond}_2(B) \leq \text{Cond}_2(A)$.

8.10

Let

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 5 \\ 1 \end{pmatrix}$$

(a) Find the unique least-square solution $x$ using

(ii) the normal equations method,

(iii) the $QR$ factorization method and the SVD method.

9.8

Applying the power method, find the dominant eigenvalue (the eigenvalue with maximum modulus) and the corresponding eigenvector for the following matrices. Do three iterations only for each matrix by choosing the initial vector randomly.

(a) $A = \begin{pmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 2 & 2 & 10 \end{pmatrix}$,  \quad (b) $A = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$

(c) A randomly generated matrix of order 4.

**QR-ITERATION Problem**

Apply three iterations of the $QR$ iteration algorithm to each of the matrices of **Problem 9.8** above and compare the eigenvalues obtained in each case with those obtained by MATLAB function `eig`. 