Theoretical Assignment 3
DUE ON SEPTEMBER OCTOBER 3, 2012 Chapter 4

No. 4.0
Answer “True” or “False” to the following. Give reasons for your answers.
(a) If a backward stable algorithm is applied to a computational problem, the solution will
be accurate.

(b) A backward stable algorithm produces a good approximation to an exact solution. (c)
Well-conditioning is a good property of an algorithm.

(d) Cancellation is always bad.

(e) If the zeros of a polynomial are all distinct, then they must be well-conditioned.

(f) An efficient algorithm is necessarily a stable algorithm.

(g) Backward errors relates the errors to the data of the problem.

(h) A backward stable algorithm applied to a well-conditioned problem produces an ac-
curate solution.

(i) Stability analysis of an algorithm is performed by means of perturbation analysis.

(j) A symmetric matrix must be well-conditioned.

(k) If the determine of a matrix $A$ is small, then it must be close to a singular matrix.

(l) One must perform a large amount of computation to obtain a large round-off error.

No. 4.1
(a)
Show that the floating point computations of the sum, product, and division of two numbers
are backward stable.

No. 4.2
Are the following floating point computations backward stable? Give reasons for your answer
in each case.
(a) $fl(x + 1)$
(b) $fl(x(y + z))$

No. 4.3
Identify which of the roots of the following polynomials are ill-conditioned and give reasons
for your answers.
(a) $x^3 - 3x^2 + 3x - 1$.
(b) $(x - 1)^3(x - 2)$.
(c) $(x - 1)(x - 0.9999)(x - 2)$.

No. 4.4
Work out the flop-counts for the following simple matrix operations

(i) Multiplication of matrices $A$ and $B$ of orders $n \times m$ and $m \times p$, respectively.

(vi) Computation of the matrix $A = \frac{uu^T}{u^Tv}$, where $u$ and $v$ are $m$ column vectors.

(vii) Computation of the matrix $B = A - uv^T$, where $A$ and $B$ are two $n \times n$ matrices and $u$ and $v$ are two column vectors.

No. 4.5
Develop an algorithm to compute the following matrix products. Your algorithm should take advantage of the special structure of the matrices in each case. Give flop-count and show storage requirement in each case.

(a) $A$ and $B$ are both lower triangular matrices.

(b) $A$ is arbitrary and $B$ is lower triangular.

No. 4.6
A square matrix $A = (a_{ij})$ is said to be a **band matrix** of bandwidth $2k + 1$ if

$$a_{ij} = 0 \quad \text{whenever} \quad |i - j| > k.$$ 

Develop an algorithm to compute the product $C = AB$, where $A$ is arbitrary and $B$ is a band matrix of bandwidth 3, taking advantage of the structure of the matrix $B$. Overwrite $A$ with $AB$ and give flop-count.