4.11
(a) Show that if \( f(x) = \log(x) \), then the condition number, \( c(x) = \frac{1}{\log(x)} \).
(b) Using the above result (or otherwise), show that \( \log(x) \) is ill-conditioned near \( x = 1 \).

4.16
(a) How are \( \text{Cond}_2(A) \) and \( \text{Cond}_2(A^{-1}) \) related?
(b) Show that
   (i) \( \text{Cond}(A) \geq 1 \) for a norm \( \| \cdot \| \) such that \( \| I \| \geq 1 \);
   (ii) \( \text{Cond}_2(A^T A) = \{\text{Cond}_2(A)\}^2 \);
   (iii) \( \text{Cond}(cA) = \text{Cond}(A) \) for any given norm.

4.17
(a) Let \( A \) be an orthogonal matrix. Then show that \( \text{Cond}_2(A) = 1 \).
(b) Show that \( \text{Cond}_2(A) = 1 \) if and only if \( A \) is a scalar multiple of an orthogonal matrix.

4.18
Let \( U = (u_{ij}) \) be a nonsingular upper triangular matrix. Then show that

\[
\text{Cond}_\infty(U) \geq \frac{\max |u_{ii}|}{\min |u_{ii}|}.
\]

Hence construct a simple example of an ill conditioned nondiagonal symmetric positive definite matrix.

4.19
Let \( A = LDL^T \) be a symmetric positive definite matrix where \( L \) is a unit lower triangular matrix and \( D = \text{diag}(d_{ii}) \). Then show that

\[
\text{Cond}_2(A) \geq \frac{\max(d_{ii})}{\min(d_{ii})}.
\]

Hence construct an example for an ill-conditioned nondiagonal symmetric positive definite matrix.

4.20
Prove that for a given norm, \( \text{Cond}(AB) \leq \text{Cond}(A)\text{Cond}(B) \).