Chapter 5
Computer Assignment 2
due on October 25, 2012

M5.1
Based on Algorithm 5.1, write a MATLAB program, called \texttt{lugewp}, to compute $L$ and $U$ such that $A = LU$ and the associated growth factor $gf : [L, U, gf] = \texttt{lugewp}(A)$.

Test data;

(1) $A = \begin{pmatrix} 10^{-15} & 1 \\ 1 & 1 \end{pmatrix}$, (2) $A = \begin{pmatrix} 1 \\ 0.0001 & 1 \end{pmatrix}$, (3) $A = \begin{pmatrix} 10 & 1 & 1 \\ 1 & 10 & 1 \\ 1 & 1 & 20 \end{pmatrix}$

(4) the matrix $A$ in (5,7) with $n = 10$
(5) $A = 20 \times 20$ Hilbert matrix
(6) the matrix $A$ in (5,8).

Print in each (i) $\frac{\|L\|_F \cdot \|U\|_F}{\|A\|_F}$, (ii) $\frac{\|A-LU\|_F}{\|L\|_F \cdot \|U\|_F}$, (iii) $\frac{\|A-LU\|_F}{\|A\|_F}$, and (iv) the growth factor.

Write your observations.

M5.2
Based on Algorithm 5.2, write a MATLAB program, called \texttt{lugepp}, to compute (i) $P, L$ and $U$ such that $PA = LU$, using partial pivoting, and (ii) the associated growth factor $gf$:

$[L, U, P, gf] = \texttt{lugepp}(A)$.

Print $\frac{\|L\|_F \cdot \|U\|_F}{\|A\|_F}$, $\frac{\|PA-LU\|_F}{\|L\|_F \cdot \|U\|_F}$, and the growth factor for each of the matrices $A$ of Problem M5.1. Explain why these results are different.

M5.5
(Experiment on the growth factor for GEPP.) Plot the growth factors for GEPP of 500 randomly generated matrices of varying dimension. Write down your observations.

M5.6
Random triangular matrices usually become more and more ill-conditioned as the dimensions increase. However, the lower triangular matrices $L$ from $LU$ factorization of a matrix $A$ using GEPP are believed to have low condition numbers. Perform an experiment to verify this statement, as follows: Take a random matrix of order 125 and compute its LU factorization using \texttt{lugepp} and plot the entries of the inverse of $L$ with entries of magnitudes greater than or equal to 1. Then change the signs of the subdiagonal entries of $L$ randomly to create another lower triangular matrix $\tilde{L}$ and plot the entries of the inverse of $\tilde{L}$ with entries of magnitudes greater than or equal to 1. Then compute $\max_{i,j} |(L^{-1})_{ij}|$ and $\max_{i,j} |(\tilde{L}^{-1})_{ij}|$. Repeat the above experiment with random matrices with entries uniformly distributed in $[-1, 1]$. 