Computer Homework 2

DUE: MARCH 19, 2013

1. Write MATLAB a function in the following format to implement Newton’s methods with divided differences.

```
function \[YI, DD, PC\] = newintdd \(X, Y, n, XI\)
```

**Inputs:**
- \(X\) – The vector containing the abscissas of the data
- \(Y\) – The vector containing the ordinates of the data
- \(n\) – degree of the interpolating polynomial
- \(XI\) – The vector containing the abscissas of the interpolating points

**Outputs:**
- \(YI\) – The vector containing the interpolated values
- \(PC\) – The vector containing the coefficients of the interpolating polynomial
- \(DD\) – The diagonal entries of the divided differences table
- \(FD\) – The diagonal entries of the forward differences table

Data: Interpolate \(f(x) = \ln(x)\), \(1 \leq x \leq 4\), with \(h = 0.5\) at \(x = 1.9, 2.9\)

2. (Experiment with the Runge Function). [The purpose of this experiment is to demonstrate the fact that the approximations obtained for the Runge function using the Chebyshev nodes and cubic splines are much more accurate than those obtained by the higher-order single interpolating polynomials.]

(a) Interpolate \(f(x) = \frac{1}{1 + 25x^2}\) using Newton polynomials of both degrees 5 and 10 based on equally spaced nodes over \([-1, 1]\).

   Draw the graph of \(f(x)\) and those of the interpolating polynomials in a single plot.

(b) Repeat step (a) with the Chebyshev nodes (that is, with the nodes at the zeros of the Chebyshev polynomial in \([-1, 1]\)).

(c) Interpolate using a cubic spline with 11 equally spaced nodes. (Use MATLAB functions `interP1`).
(d) Prepare a table of interpolation errors in the following format with $x = -1 : 0.2 : 1$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>Newton $P_{10}(x)$</th>
<th>Chebyshev $P_{10}(x)$</th>
<th>Spline $(x)$</th>
<th>Newton Error</th>
<th>Chebyshev Error</th>
<th>Spline Error</th>
</tr>
</thead>
</table>

\[ \begin{align*}
\text{Newton } P_{10}(x) & \quad \text{The value of the 10th degree Newton interpolating polynomial at } x \text{ with standard nodes} \\
\text{Chebyshev } P_{10}(x) & \quad \text{The value of the 10th degree Newton interpolating polynomial with Chebyshev nodes} \\
\text{spline } (x) & \quad \text{The value of the spline function at } x
\end{align*} \]