HWK1
Due : February 7, 2013

1. Suppose that the bisection method is used to find a zero of \( f(x) = x^3 - 7x + 6 \) in \([0.5, 2]\) with an accuracy of \(10^{-5}\). How many minimum numbers of iterations will be needed? Do this number of iterations to verify the statement. Present your results in a tabular form containing approximate value of the zero at each iteration.

2. Derive an algorithm to compute \( \sqrt[5]{A} \) based on Newton’s method and apply your algorithm to compute \( \sqrt[5]{30} \), by choosing a convenient initial approximation and then run as many iterations as needed to obtain a convergence of 4 decimal place accuracy. What is your observation about the rate of convergence?

3. Repeat the problem No 2 with the secant method by choosing \( x_0 \) as above and another suitable approximation \( x_1 \). Establish empirically that the rate of convergence this time is linear (Do as many iterations as necessary to arrive at your conclusion).

4. (a) Sketch a graph of \( f(x) = x + \ln x \) and find an interval containing a zero \( \xi \) of \( f(x) \). Show that the related iteration \( x_{i+1} = g(x_i), x_0 \) is given, \( x_0 \neq \xi \), with \( g(x) = -\ln x \), does not converge to \( \xi \) even if \( x_0 \) is arbitrary close to \( \xi \).

(b) Now, select an iteration function so that the corresponding fixed point iteration will lead to convergence for any choice of \( x_0 \) in the interval containing the root. Do five iterations with this iteration function choosing a suitable initial approximation.

5. The iteration \( x_{n+1} = 2 - (1 + c)x_n + cx_n^3 \) will converge to \( \xi = 1 \) for some values of \( c \) (provided \( x_0 \) is chosen sufficiently close to \( \xi \)). Find the values of \( c \) for which this is true. For what value of \( c \) will the convergence be quadratic?

6. The polynomial \( P(x) = x^7 - 2x^6 + x^5 + 5x^3 - 3x^2 - 9x + 7 \) has a double root at \( x = 1 \). Approximate this root first by using the standard Newton method and then by two modified Newton’s methods for multiple roots, starting with \( x_0 = 0.5 \). Use Horner’s scheme to find the polynomial value and its derivative at each iteration. Do three iterations for each of three methods and present your results in tabular form containing (i) the approximate value of the root at each iteration, (ii) corresponding functional value, and (iii) the ratio of the error at each iteration to that of the previous iteration. What is rate of convergence? Give reasons for your answer. (Prepare one table for each method).