1. Derive the error formula for the standard Simpson’s formula and using it prove the error formula for the composite Simpson’s rule:

\[ E^s_c = -\frac{(b - a)}{180} h^4 f^{(4)}(\eta), \]

where \( a \leq \eta \leq b \). (Consult your textbook, if necessary).

2. Suppose that it is required to estimate \( \int_0^1 e^{-x^2} \, dx \) within an accuracy of \( \epsilon = 10^{-5} \). Find \( h \) using

(a) The composite trapezoidal rule

(b) The composite Simpson’s rule
3. Let $P_n(x)$ be the $n$th Legendre polynomial and $Q(x)$ be any polynomial of degree less than $n$. Then prove that
\[ \int_{-1}^{1} Q(x)P_n(x)\,dx = 0. \]

4. Approximate $\int_{1}^{5} \frac{dx}{x + 5}$ using Gaussian Quadrature with $n = 2$ and $n = 3$. 
5. Prove that the Quadrature formula of the form
\[ \int_1^3 f(x)dx \approx c_1 f(1) + c_2 f(2) + c_3 f(3) \]
that is exact for all polynomials of as high a degree as possible, is nothing but Simpson’s rule.

6. Use adaptive Quadrature rule to approximate \( \int_1^3 \frac{1}{x} \, dx \) to within \( \epsilon = 10^{-3} \).
7. Suppose that Romberg integration is used to approximate

\[ \int_{0}^{1} \frac{x^2}{1 + x^3} \, dx. \]

Develop a Romberg table with \( n = 3 \), and use this table to compute an approximation of the above integral.