A FOURTH-GRADE TEACHER USES A
CONSTRUCTIVIST APPROACH TO MATHEMATICS AND TEACHING AND LEARNING

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Abstract
What happens when an elementary teacher interprets and implements a constructivist approach to mathematics learning and teaching? Reshaping mathematics teaching based on a constructivist view of learning presents a considerable challenge to an elementary teacher. This study describes one teacher’s approach to this theory in practice. The researcher, as a participate observer in this classroom, used ethnographic research methods to collect data for 4 1/2 months during each mathematics class. Data analysis revealed 10 instructional strategies the fourth-grade teacher simultaneously used. Three specific teaching strategies during classroom vignettes are discussed in detail in this article.
Mathematics teachers, teacher educators, and researchers involved in the current reform movement in mathematics education suggest major changes in the teaching of mathematics. They recommend that students become actively involved in constructing their own knowledge and developing mathematical concepts as they explore, explain, and justify solution strategies to mathematical tasks. Guidelines for helping students construct their own mathematical knowledge are provided in *The Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and *The Professional Standards for Teaching Mathematics* (NCTM, 1991). These documents deemphasize accuracy and memorization of computational procedures and emphasize teachers helping students develop mathematical power. The NCTM (1989) has defined mathematical power as “the individual’s ability to logically reason, conjecture, and effectively communicate about mathematics” (p. 6). Teachers help students construct mathematical power by teaching in contexts of problem solving and reasoning about mathematics through engaging them in communicating about the processes they use to reach solutions. For many teachers, this “constructivist approach” to mathematics teaching requires a change in their conceptions about mathematics and about what it means to learn and teach mathematics (Steele & Widman, 1997).

**Theoretical Framework: Constructivist Learning Theory**

How do constructivist learning principles guide teaching? What is a constructivist teacher? Answers to these questions are of concern to mathematics educators. Although several researchers have begun to study what constructivist teaching involves (Fennema & Nelson, 1997; Schifter & Fosnot, 1993; Simon & Schifter, 1991; Cobb, Yackel, & Wood, 1990, 1992; Kamii, 1989, 1995), few models of this type of teaching exist.
Although constructivist learning theory (constructivism) provides mathematics educators with useful ways to understand how individuals construct their own knowledge, the task of reshaping mathematics teaching based on a constructivist view of learning presents a considerable challenge. Constructivism does not tell us how to teach mathematics. Cobb (1988) has said that a teacher with constructivist views facilitates students’ construction of knowledge by negotiating with the students during interactions in the classroom. Therefore, a “constructivist teacher” is one who provides active experiences for students that encourage students to reason and communicate that reasoning. This type of mathematics teaching forms the basis for this study.

According to constructivism, students do not passively receive knowledge but actively construct new knowledge based on prior knowledge. Meaningful learning requires students’ active involvement. Rather than receiving information, “the learners negotiate meaning within the context of their present understandings, make connections with past personal understandings, and modify prior knowledge in order to build new constructs” (Cobb et al., 1992, p. 6). Constructivist learning is a process by which individuals actively create or invent knowledge. Educators who advocate constructivist learning in mathematics say that children create new mathematical knowledge by reflecting on their thinking and actions while they solve problems.

From a constructivist perspective, “mathematics teaching consists primarily of the mathematical interactions between a teacher and children” (Steffe & Killion, 1986, p. 207 [as cited in Koehler & Grouws, 1992]). The teacher guides the students in communicating meaning through discourse and interaction. The students then interpret and adjust this meaning to their present mathematical understandings (von Glasersfeld, 1981). Cobb and Steffe (1983 [as cited in Koehler & Grouws, 1992]) noted that “in the constructivist view, teachers should continually make a conscious attempt to see both their own and the children’s actions from the children’s point of view” (p. 85). The important issue then becomes one of ensuring that children construct correct knowledge.
Constructivism is founded in the research of Jean Piaget. Piaget’s (1973) central concern was the process through which individuals construct knowledge. Piaget suggested that individuals actively construct knowledge internally through their actions on objects in the world and their reflections on these actions. Building up knowledge is a process of organizing and adapting to the world. This process involves modifying old knowledge to assimilate and accommodate new knowledge.

Piaget believed that people learn through a process of individually acting on the world. Piaget said that during a process of disequilibrium, individuals internally experience cognitive conflict when confronted with new information. During disequilibrium, prior knowledge cannot explain their new experiences. Therefore, through accommodating new knowledge and assimilating it with the prior knowledge, individuals form internal structures of knowledge unique to them. Piaget (1973) claimed that interactions in the classroom can help knowledge development because interaction can create cognitive conflict that can change thinking. Piaget claimed that student interactions can initiate students to individually reflect on ideas that other students present.

A recent view of constructivist learning in mathematics education research adds the work of Vygotsky (1962, 1978). According to Vygotsky (1978), individuals construct knowledge in the zone of proximal development through social interactions with more knowledgeable persons. During interactions with others, individuals come to understand their thinking and initiate changes in their knowledge. “Human learning presupposes a specific social nature and a process by which children grow into the intellectual life of those around them” (Vygotsky, 1978, p. 73). Vygotsky believed that individuals could achieve higher ground through talking; that is, through interaction individuals organize their thinking and actions. In this way, learning occurs two times for each individual, originally during social interaction and then again within the individual.

Recent researchers in mathematics education (Cobb, Yackel, & Wood, 1991; Cobb et al., 1992; Cobb, 1994) has adopted a social-constructivist approach to mathematics learning and teaching. “Students come to view mathematics as an activity in which they solve mathematical
problems by constructing personally meaningful, justifiable solutions while actively contributing to
the interaction of the group” (Cobb et al., 1991, p. 8). However, during the process of negotiating
and sharing with a knowledgeable teacher, students come to understand the mathematical
meanings of the wider society (Cobb, 1994). Students adapt their knowledge during ongoing
interactions. This approach to learning and teaching is the spirit of The Standards. The perspective
of this author is one of social constructivism and combines views of Piaget and Vygotsky and is
similar to the perspective held by Cobb et al. (1990, 1992). Both Piaget and Vygotsky have
contributed significantly to the theoretical basis of this research. It is important to balance the
cognitive and the sociological perspectives in order to understand and to promote various aspects
of learning and teaching mathematics.

Purpose

The purpose of this article is to portray how one fourth-grade teacher interprets and
implements constructivism within her mathematics teaching. This article focuses on actual
teaching episodes in this fourth-grade class. These teaching episodes demonstrate how the teacher
negotiates with her students to find and adapt activities for them to construct powerful
mathematical knowledge. These descriptions show interactions between the teacher and students
that affect the opportunities for students to learn mathematics.

This article demonstrates to inservice and preservice teachers how they may use
constructivism in their own teaching. An important step toward changing mathematics teaching is
for teachers to see or to read about alternative approaches to teaching. Through these examples,
they become more aware of problems that exist within their own teaching. Investigating real-life
teachers can help answer questions about how teachers use their knowledge and beliefs to make
decisions and provide instruction. How does a teacher decide if an answer is mathematically valid
or invalid? How can teachers understand the mathematics constructed from a child’s point of
view? Unless teachers have seen or read about alternate approaches to teaching, they will teach
content in mathematics similarly to how they were taught (Ball, 1988). By examining the practice
of teaching in actual classrooms, researchers can provide different models of what teaching “looks like” specifically when constructivism is being implemented in their practice.

Methodology

The author began searching for a teacher who was implementing a constructivist approach to mathematics learning and teaching in the classroom. She asked several administrators, teachers, and parents in several school districts to suggest elementary mathematics teachers who they believed taught mathematics effectively. For a semester, 20 different elementary classrooms were observed and 20 teachers were interviewed. During this time, a trip was made to Mustang Elementary\(^1\) to visit Barbara Clark\(^2\). Her mathematics teaching was observed on two different occasions. She questioned children and always asked them to explain their thinking. The children were actively involved in doing mathematics. They manipulated materials, drew diagrams, and worked cooperatively to solve problems. In short, here was a teacher who was helping students construct their own knowledge. Initially, she was interviewed for one hour.

Mrs. Clark was asked to complete the *Mathematics Beliefs Scales* questionnaire (Fennema, Carpenter, & Peterson, 1987) in order to assess her beliefs about how children learn mathematics, about how mathematics should be taught, and about the relationship between learning concepts and procedures. Her score on the questionnaire confirmed observations. Finally, this teacher was chosen and agreed to participate in the research.

Ethnographic research methods were used to investigate this elementary teacher while she taught mathematics. Ethnographic methods encouraged understanding of the life of the teacher within “the framework of the culture” (Bogdan & Biklen, 1982, p. 37). This qualitative methodology is composed of several characteristics (Bogdan & Biklen, 1982; Reichardt & Cook, 1979): (a) The research is conducted within the particular setting under study; (b) the researcher is the main research instrument; (c) the data are descriptive; (d) the researcher obtains meaning from the context and from the participants’ perspectives; (e) the data are analyzed inductively; (f) the

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\(^1\) Pseudonym
\(^2\) Pseudonym
research process is more important than obtained products; and (g) a holistic picture is created. Data were collected through participant observation, interviewing, and collecting artifacts. Some of the steps taken to ensure the validity and reliability of the research included collecting data over a long period of time (4 1/2 months), using several methods of data collection (interviews, observations, and artifact collection), discussing findings with participants, and keeping bias (researcher’s own views) under control.

A theoretical perspective of symbolic interactionism was assumed for this study. According to this perspective, “objects are social products in that they are formed and transformed by the defining process that takes place in social interaction” (Blumer, 1969, p. 69). Blumer’s perspective of social interaction coincides with constructivism. According to both views, mathematics would be a product of the interactions among teachers and students in classrooms. Blumer characterized symbolic interaction as a process of how a person interprets and explains the fundamentals of the situation. To understand how the teacher interpreted and understood the fundamentals of the interactions in the mathematics classroom, the researcher entered the context of the classroom environment and sought to experience the process by which the teacher made decisions. Observing the interactions among teacher and students and understanding her thinking gave me the opportunity to understand her perspectives and practices. The ethnographic research approach of participant observation, interviewing, and collecting artifacts provided a compatible methodology for investigating what happens when a teacher implements a constructivist approach to learning and teaching of mathematics in an elementary classroom.

The researcher’s actual participation in this classroom was minimal. It could be characterized as passive (Spradley, 1980). Spradley described “passive participation” of a researcher as “the ethnographer engaged in passive participation is present at the scene of action but does not participate or interact with other people to any great extent” (p. 59). As data were collected, patterns and themes began to emerge from the data analysis. These pattern and themes helped direct future observations (Bogdan & Biklen, 1982).
Sixteen hours of mathematics lessons taught by the teacher were videotaped by the researcher. All lessons were audiotaped. Both formal and informal interviews were conducted (Spradley, 1980). During short times before or after class, the teacher was asked to clarify some classroom events or episodes. For example, when she decided to choose fraction circles instead of pattern blocks to teach mixed numbers, she was asked, “Why did you make this choice?” After other lessons, examples of questions were: “Did the lesson go as you thought it would?” “Why did you change the wording of the problem?” Six formal interviews were conducted with this teacher. Sometimes during the interviews, the teacher viewed videotapes of the lessons. Some interview questions focused on how she planned her lessons. Examples of questions included the following: “How do you plan how long to devote to each concept?” “What will you do if a student cannot solve the problem?” Various artifacts were collected from the teachers’ classrooms, such as copies of pages in the teacher’s planbook, students’ work assignments, students’ portfolios, and students’ tests.

Over 1000 pages of data were transcribed. Analysis of the data followed the procedures described by Spradley (1980) as the Developmental Research Sequence. This ethnographic research model of data analysis was a cyclic process of questioning, collecting data, recording data, and analyzing data. During the research, a sequence of questioning, collecting, recording, comparing, contrasting, and analyzing was continually repeated. Data analysis was an integral part of the research cycle.

**Findings: Mrs. Clark’s Instructional Strategies**

Mrs. Clark’s teaching is a balance between attention to conceptual understanding and application. The students engage in problem situations by identifying relationships and patterns while they search for answers. The teacher guides the students to discover mathematical rules for themselves by asking them to present and to justify the solutions that they create for problems or algorithms. She sometimes provides clues during problem solving to guide her students toward
either concepts or procedures. She asks the students to apply their learning to new situations. She
demonstrates that learning to think can be engaging, enjoyable, and empowering.

Through observations and analyses of the data, 10 instructional strategies were identified that
demonstrate Mrs. Clark’s implementation of constructivism, however, Mrs. Clark did not
consciously use these strategies. In order to understand and to appreciate the whole quality of
Mrs. Clark’s teaching, one must integrate all the strategies. In other words, her model of teaching
is the combination of all her strategies. Nevertheless, in order to analyze Mrs. Clark’s teaching
more subtlety, the strategies needed to be separated. Three of the strategies will be discussed in this
article.

Uses Concrete Problem-Solving Contexts

Central to Mrs. Clark’s mathematics teaching are the problem-solving experiences she
provides for her students. The problem-solving contexts include a variety of word problems,
strategies for solving problems, problems using manipulatives to model thinking, open-ended
problems, and everyday real-world problems. In these way, Mrs. Clark helps students to see that
mathematics is everywhere. This teacher does not teach skills to be learned for problem solving
but teaches skills through problem solving. In her teaching, problem solving is not a distinct topic
separate from other mathematics learning. Hiebert and Carpenter (1992) wrote,

The implication is that learning situations should be embedded in authentic problem
situations that have a meaning for the students. By learning concepts and
procedures in problem-solving contexts, it is presumed that the knowledge is
connected so that it is accessible for problem solving. (p. 81)

Ms. Clark organizes most classes around open-ended nonroutine problems that have no
clear paths to solutions and no ready algorithms. Most problems have several solutions and several
ways to find solutions. She often uses her own life experiences as a resource in planning problem-
solving situations for the students to investigate. For example, she made up a problem from her
experience one weekend when she and her family had taken their RV on a camping trip. On their
way they had stopped at an RV supply store to get some items. When she was ready to check out, a man in front of her was completing his purchase. He said to the clerk, “Why, you gave me back the same number of bills that I gave you.” His purchase had been $282.64. She said “I thought, what a great problem! I am going to tell my students the amount he purchased and see if they can come up with which possible bills he gave them and they gave back to him.” This was the problem that she ended up asking:

A man purchased some material at a store. The total cost was $284.64. He gave the clerk $300.00. Upon receiving his change, he said, “I got the same number of bills back that I gave you.” What bills did he give the clerk? What bills did he get back?

Mrs. Clark involves students actively in the learning process. Von Glasersfeld (1981) wrote that active learning is taking new information and acting upon it in some way to create meaning. Students’ experiences of learning the content help them understand that mathematics is a process of learning to think. In this class, students do not sit passively and watch the teacher perform actions to solve the problems but they solve the problems themselves and become involved in learning mathematics. Even though Mrs. Clark’s mathematics class usually lasts from an hour to an hour and a half, often students became so involved that they asked to skip recess so that they could continue with their mathematics.

Mrs. Clark never hurries through problems but concentrates each day on a few significant problems. Many class periods, students will discuss only three or four problems. Mrs. Clark’s problems have a clear purpose. Her purpose for each day makes each lesson or experience focused and coherent. The problems demand conceptual knowledge and higher-order thinking strategies from her students. She emphasizes primary concepts that are central principles that define mathematics; for example, addition and subtraction are inverse operations; a fraction is a part of a whole; a fraction's value depends on the whole to which it refers; and a circle has only one center.
Brooks and Brooks (1993) emphasize the importance of providing learning around primary concepts.

Mrs. Clark does not sequence her instruction from part-to-whole. She does not break the problems down into steps for the students. She presents the problems or activities in a conceptual context where students learn to look at the whole conceptual relationship or meaning of the problem. Mrs. Clark guides the students to look at the whole in order to break it into parts. She does this so that students understand the parts they have created and how the parts relate to the whole concept.

For example, the students had worked independently on the following problem since they and gotten to school at 7:45 a.m.:

Is it possible to make a solid figure on a geoboard? If yes, explain how. If no, explain why not.

On several days before this question, the students had learned about relationships between two-dimensional and three-dimensional figures. For example, they had discussed the shapes of faces of cuts on cylinders, pyramids, prisms, spheres, and cones.

At 8:00 of this day, the following discussion took place,

Mrs. Clark: Pat, what do you think?
Pat: No. Because if you put rubber bands on there it is not going to be a block.

Mrs. Clark: What does being like a block have to do with being a solid figure?
Pat: Well, if you want a solid figure, you have to make a block. Or then it is just plane. It is just a plane figure.

Mrs. Clark: Anybody else? Jim? (Many students are saying, “Oh. Oh. Oh.” with their hands up.)

Mrs. Clark prods and probes to get students to contribute their ideas and clarify their thinking. She says, “Tell me more.” “What do you think?” Other students answer,
Jim: No, if it is a solid figure, there’s not going to be air in it. You can’t do that on the geoboard.

Denny: No, because you can’t make the sides go up here on the geoboard and hold it in your hand.

Eddie: On a geoboard, it is flat, and it’s not three-dimensional.

Mrs. Clark: So you are saying, “No, because in order for it to be a solid figure, it cannot be flat. It has to be three-dimensional?” Now, I think I have made a mistake. I want to change one of my words up there. I want to say, “Is it possible to represent a solid figure on the geoboard?” Maybe that makes a difference in your thinking--maybe not. All right, Mary?

Mrs. Clark summarizes the students’ reasons and decides to change the way she has worded the problem. By rewording the problem, she stimulates students’ thinking.

Mary: Yes, because if you just keep on stacking them around. It won’t be flat anymore. You’ll have more rubber bands and you can fill the whole thing in?

Mrs. Clark: When we talk in mathematics about a solid figure, does that mean it has to be a completely filled in space?

Students: Yes.

Dan: Because if it goes up and up and up, it’s not flat. I stacked my rubber bands up from the bottom of the nail to the top to make it not flat.

Mrs. Clark: To make it what?

Students: Solid.

Eddie: Three-dimensional.
Mrs. Clark: It makes it three-dimensional. Danny’s figure has length, width, and height. Look at Sharon’s. She has got something other than a square shape represented. She has a triangular shape. Hal also has a triangular shape represented that has length, width, and height.

Mrs. Clark highlights Eddie’s answer by saying what three-dimensional means. Then, she uses other students’ representations on the geoboard to push the point. She has observed Ann’s representation and asks Ann to hold up her geoboard. She has used rubber bands to make her figure look like this (see Figure 1).

![Figure 1. Ann’s Geoboard Solid Figure.](image)

Mrs. Clark: What shape does that represent?

Students: Cube.

Mrs. Clark: A cube. It does look three-dimensional like that, doesn’t it? All right, raise your hand if you think now that after listening to these people’s explanations that the answer is no. (Students’ hands up.) Raise your hand if you think that he answer is yes. (Students’ hands up.) If you raise your hand and say yes, I want you to show me on the geoboard. Make sure that what you are showing me represents a solid figure.

Although the context of this lesson is geometry, this problem is not just about definitions of solid shapes. The students were engaged in discussing and making a distinction between a three-dimensional object and its representation on a flat two-dimensional surface. Thinking about building the solid figure on a two-dimensional surface presented the students with a challenge, but
this activity was an informal way for the students to get to know and understand a solid shape. Through listening to the students, the teacher realized that she needed to change the word make to represent. She changed only one word to get them to think deeper. At the conclusion of the activity, Eddie, one of the students, decided that he would answer the question with a yes only if the word represent were used. He was adamant that if the word make was used, the answer would have to be no. He said that Ann’s representation of the cube on the geoboard had convinced him.

Mrs. Clark did not circle the answer, yes or no. Mary said, “Mrs. Clark, you never did circle the answer up there, yes or no.” Mrs. Clark said, “You are right, I didn’t. I wonder if there is a reason that I did that?” Mary answered, “Some people said yes, and some said no.” Mrs. Clark responded, “So we didn’t reach a definite answer.” Many times in Mrs. Clark’s class there was not always resolution about a final answer to her questions or problems, but she used the unresolved opportunities to build future lessons. On following days, the students had opportunities to construct three-dimensional drawings on isometric dot paper in order to develop understanding of spatial perspective.

Builds Instruction on Students’ Prior Knowledge

Mrs. Clark guides students to connect their past learning experiences to their present and future learning. She values the students’ prior knowledge. Mrs. Clark provides in-class experiences that relate to her students’ lives outside of mathematics class. She stimulates their interest so that they make sense of their past learning experiences. This teacher believes that students have valuable intuitive, informal knowledge that is important for them in understanding mathematics.

Mrs. Clark observes her students solve problems and listens to their strategies so that she can obtain information about their conceptual understanding. She then builds her instruction upon the students’ existing conceptual knowledge. She says, “I need to find where they are building
knowledge. I am hoping to add blocks to their building. It may be at different levels. They come to me with a certain amount of knowledge. I am hoping to add to that.”

In the following probability problem, Mrs. Clark uses the student’s intuitive knowledge of probability to scaffold the students to develop the procedures for writing ratios for probability. Bruner (1986) used the word “scaffolding” as the process by which a competent adult helps those less competent gain a higher level of thinking. According to Lampert (1991), “beginning a problem in a familiar domain . . . is a way that teacher and students can communicate in what Vygotsky (1962) called the ‘zone of proximal development’” (Lampert, 1991, p. 127).

Mrs. Clark helps students develop their own procedural algorithms. Kamii (1989) noted that when children invent their own algorithms, they become more competent than when the algorithms are told to them. Kamii stated that “procedures children invent are rooted in the depth of their intuition and their natural ways of thinking” (p. 14). With this lesson as with many others, Mrs. Clark has helped students connect their intuitive knowledge to their new knowledge.

The problem was:

In a sack of 30 coins, the probability of drawing out pennies is 1 out of 2. A nickel has a 2 out of 5 probability of being drawn. A dime has 1 out of 10 chance of being drawn. There are ____ pennies, ____ nickels, and ____ dimes in the sack.

Mrs. Clark: How many pennies do you think are in the sack, John?

John: 15 pennies.

Mrs. Clark: Why do you think 15?

John: Because half of 30 is 15.

Mrs. Clark later said that she was surprised that John gave the correct answer so quickly. This year is the first time she has formally taught probability to fourth graders, so she was uncertain about where their knowledge would begin. She knew, however, that it was important for her find out where to begin instruction. After John’s answer, the students discuss more about why there
might be 15 pennies in the sack. Using John’s answer, Mrs. Clark’s writes on the chalkboard 1/2 and 15/30 beside each other.

\[
\frac{1}{2} \quad \frac{15}{30}
\]

She asks the students if they think any connection might be made between the two fractions. Jerry says he thinks they are equal. Others contribute their opinions. Mrs. Clark finally puts the symbol, =, between the two fractions (1/2 = 15/30). She also writes 15 pennies in the blank in the question. They begin a discussion about how many dimes are in the sack. Many students have guesses. Most are incorrect. Eddie says that the 1 out of 10 probability should be 1/10. Mrs. Clark writes this on the board.

\[
\frac{1}{10}
\]

Suddenly Jim raises his hand and says excitedly,

Jim: Could you do the same thing (the same thing they did to get 1/2 = 15/30) to that (he points to the 1/10) and see what numbers go into the top and bottom numbers?

Mrs. Clark: All right. We are going to do the same thing here. What is the denominator going to be?

Some students: 30.

Mrs. Clark: What is the numerator going to be?

Ken: 20. (There were no other suggestions from students. So Mrs.

Clark writes 20/30 on the chalkboard.)

Even though Mrs. Clark knows this answer is incorrect, she writes 20/30 on the chalkboard.

Mrs. Clark: Dan, can you help us out?

Dan: Ten times 3 is 30. So it would have to be 1 times 3 is 3. (Mrs.

Clark writes \(\frac{1 \times 3}{10 \times 3} = \frac{3}{30}\) on the chalkboard.)
Mrs. Clark: Okay. Did everybody see what Dan did? Ten times 3 is 30 and 1 times 3 is 3. What does this tell us?


Mrs. Clark: What else does it tell us? Dan?

Dan: That there is [sic] 3 dimes.

Mrs. Clark: All right. Dan is saying that there are going to be 3 dimes.

Most of the students agree with Dan, so Mrs. Clark writes 3 in the blank for dimes on the problem. With several students’ contributions, Mrs. Clark builds with nickels, in the same way as with the dimes, the new ratio of 2/5 = 12/30. However, some disagreement develops. Joan provides a reason why she does not think that 30 should be in the denominator for the fraction.

Joan: If we are at 2/5, wouldn’t you start with the denominator at 15 because 15 of the coins are already gone?

Mrs. Clark: That’s a good question. Joan is suggesting that the denominator should be changed to 15 because we have decided that there are 15 pennies already gone.

A Student: Say it again.

Mrs. Clark: What Joan thinks is that the denominator should be 15.

Her question was--since we already used 15 pennies, shouldn’t there only be 15 coins left?

Hal: No!!! (He is adamant with his statement.)

Mrs. Clark: Hal is saying no to you, Joan. Why do you say, no, Hal?

Hal: Because the (does not remember what term to use) top number

Mrs. Clark: Numerator

Hal: Of 2/5 is 2. The numerator of 1/10 is 1, so 3/30. So 12/30. If you add 12 and 3, it equals 15 and that is the other half for the 15 (pennies). 15 + 15 would equal 30.
Mrs. Clark: Hal is saying that we need all the same denominators. When we add the numerators—we have $15 + 3 + 12$ and that is 30. (Mrs. Clark writes the following on the chalkboard.)

$$15 + 3 + 12 = 30$$

Mrs. Clark took John’s answer of 15 and his explanation that half of 30 is 15 and built this day’s lesson around it. Mrs. Clark said later that she was very pleased with what happened. She said, “I think they know some probability intuitively. The reason they began to have trouble later is that they are required to think about it in certain procedural ways that are not intuitive for them.” The discourse from this lesson demonstrated that some students were already thinking about the difference in the chances of drawing out certain coins within independent and dependent events.

The research of others (Fischbein, 1975, 1991; Leinhardt, 1988; Mack, 1990) has emphasized the importance of intuitive knowledge in children—non-schooled knowledge that students hold without immediately conscious reasoning. Mack (1990) characterizes intuitive knowledge as “applied, real-life circumstantial knowledge constructed by the individual student” (Mack, 1990, p. 16). Fischbein (1991) states that a central issue in learning probability are the natural, intuitive approaches that individuals hold. Intuitions are intrinsic to reasoning (Fischbein, 1975). Understanding student’s intuitive knowledge is important before students learn algorithms because intuition is a bridge on which to build. Mrs. Clark builds on students’ intuitive knowledge so that they will not simply memorize procedures for solving problems that they do not understand. Kamii (1989) suggested that “if we encourage [students] to develop their own ways of thinking rather than requiring them to memorize rules that do not make sense to them, children develop a better cognitive foundation [for the procedures]” (p. 14).

Looking back at this lesson, we can see how Mrs. Clark guided students toward conceptual understanding of probability. The lesson helped them construct concepts about ratio and proportion. The discussion showed how she built upon the explanation of one student, John, in knowing that $1/2$ of 30 should be 15, to lead the discussion to complex concepts for fourth
graders. As Mrs. Clark recorded their ideas and provided a representation of their reasoning, she gave her students the opportunity to connect their conceptual knowledge to the procedures (NCTM, 1991). Mrs. Clark was also attempting to scaffold the students intuitive probabilistic thinking at an informal level in order to acquaint them with mathematical proportions. Lamon (1994) suggests that students naturally use mathematical intuition. She states “student representations of situations involving ratio and proportion occur on an informal . . . level long before those students are capable of treating the topic quantitatively . . . “ (p. 99). The research of others (Karplus, Pulos, & Stage, 1983; Post, Behr, & Lesh, 1988) propose that the development of proportional reasoning in children is complex.

Helps Students Construct Vocabulary While Learning to Use It

One of the ways that Mrs. Clark helps her students construct a knowledge base for thinking about mathematics is by helping them learn and use the “language of mathematics.” Mrs. Clark thinks that vocabulary in mathematics is important. She teaches students how to think, speak, and write mathematically. She provides correct mathematical terms. Duckworth (1987) suggested that we give children “language tools” not only to facilitate clearer thinking but also to help them to communicate their thinking to others. When children use correct terms, we can pay more attention to what is being said rather than how it is being said. When I asked Mrs. Clark why she emphasized vocabulary in her teaching, she responded,

I think hearing big words used regularly makes them less afraid of them when they have to know them. We don’t really get to the point of studying terms like 90 degree angles in the fourth grade. But if I can slip that in every now and then, when they have to know them in later grades, it’s not a foreign idea. [For instance] when I get to fractions, I don’t have to teach what is a numerator and denominator because I’ve already used it and used it in a way they know that they are not going to be tested. They have some prior knowledge with it.
Mrs. Clark usually presents new vocabulary words in a context in which the students are studying something familiar. One morning Mrs. Clark asked the question,

If you have a circle drawn on your paper, how can you find the center of the circle if you don’t already know where it is?

The students enjoyed investigating and solving this problem. Students began exploring the relationships involved in finding the center of a circle and creating solutions while they learned mathematical vocabulary. In this problem, Mrs. Clark has emphasized one of the primary concepts central to mathematics, a circle has only one center. Mrs. Clark has asked her students to think about the whole conceptual relationship of what the center of a circle means while constructing the parts.

The students are excited to share their solutions. Mrs. Clark has asked Gerry to go up to the chalkboard and show how she found the center of the circle. Gerry takes a plastic lid, draws a circle, then uses a ruler to draw a radius. Mrs. Clark asks her to tell what she is thinking as she draws. Gerry says, “Take the ruler and find a point that you think is the center and mark it. Then move the ruler around and mark it and keep doing it.” Gerry measures from the point that she has marked as the center to the edge of the circle and gets 5 inches. She keeps drawing radii (see Figure 2). The discussion continues.

![Figure 2. Gerry’s Drawing](image)

Mrs. Clark: Why do you think that it was important to have 5 inches or how did you choose where you started?

Gerry: Because it looks like it is about the middle.

Mrs. Clark: She is picking a point. Guessing where the center might be.

Looking at it and moving it around to see if it stays the same at 5.
But look around here, Gerry, we have a problem. It’s over 5 there. (Gerry tries to adjust for this by measuring again and checking the distance to the edge of the circle in several places.)

Mrs. Clark: What’s happening to the other end of your ruler, Gerry? Where your right hand is?

Gerry: It’s going out a little.

Mrs. Clark: Is your left hand slipping? Is it hard to hold the same?

Gerry: Yes.

Mrs. Clark: Okay. We could do it that way. The problems we have are that we have to adjust and it is hard to hold the ruler in the same place as it goes around. If we worked at it we could probably find the center, couldn’t we? Another suggestion. Ken? (Many students want to show their method.)

Ken: I think it would be better if you found half of it.

Ken goes up to the chalkboard, picks up the ruler, and turns it around so that the end with 1 inch is on the edge of the circle. Mrs. Clark asked him why he changed ends. He says, “Because I had the 12 up there and I needed to have the 1.” Ken gets 9 for the width of the circle. He says, “It’s 9. So half of 9 is 4 1/2.” The discussion continues,

Mrs. Clark: Let’s think about what Ken did. When he measured from one point on the edge of the circle to another point on the edge of the circle, what was he measuring for?

Students: Diameter. (Some students say the diagonal, but most say it correctly.)

Ken draws that diameter. Then he measures and draws another one (see Figure 3).
Mrs. Clark: Ken, why did you use another diameter?
Ken: To see if they are the same.

Mrs. Clark: Is that important? What if he had it too far down like this. And Ken thought it was here. Is that going to make a difference? (Mrs. Clark draws (see Figure 4) a line lower in the circle that looks like a chord of the circle.)

Students: Yes.

Mrs. Clark goes on to tell the students that this line segment is called a chord and would not be considered a diameter.

Mrs. Clark: What happens to the circle as you get away from the center?
Denny: It curves and goes--it curves like this and goes down. (He uses his hands to show a smaller distance.) So that way the sides of the circle get smaller.

Mrs. Clark: The distance between points as we get away from the diameter gets smaller. Would you say that the diameter is the widest part of the circle?

Students: Yes.

Mrs. Clark: So he checked two places to see if the diameter was the same.

Then you did what, Ken?

Ken: I halved 9 which is 4 1/2 and I marked it in the middle.

Mrs. Clark: The distance from the point which he is thinking might be the center to the edge of the circle is called the radius.
Mrs. Clark has given students the opportunity to more accurately understand the meaning of diameter and has provided them with two new words, radius and chord.

Ken then measures his second diameter to see if they are the same length. They are not. Mrs. Clark asks if someone would like to help him. Hal goes up to the chalkboard to show how he would do it. Hal measures draws six diameters and measures them to be sure they are the same length (see Figure 5).

![Figure 5. Hal’s Drawing of Six Diameters.](image)

Mrs. Clark: And what are they supposed to do in the center?

Hal: All of them are supposed to go through the middle and cross.

Mrs. Clark: When they cross in the center of the circle, they intersect. How many diameters do you think that you have to draw, Hal?

Till you decide that is the center?

Hal: (counts) 6.

Mrs. Clark: So you think that in order to find the center, we need to draw exactly 6 diameters? You couldn’t do it with 5 or 4?

Hal: No, cause it wouldn’t be enough.

Mrs. Clark then asks the whole class how many would be the least number of diameters that he could draw to check the center. The students continue with this discussion for about 15 minutes. Several other students show their solutions. One student uses a protractor and one cuts out a circle and folds it in half and then into fourths. After several arguments, most students decide that there should be at least two.

Mrs. Clark has given students the opportunity to articulate their thinking and learn vocabulary in an open-ended problem-solving context. She believes that understanding comes from crossing the same territory in many different directions. The process of having students
explain their solutions is repeated again and again. The students learn to use the language of mathematics and become very articulate at expressing their thinking.

Mrs. Clark has involved students in a problem that has helped them understand how to draw plane figures, in this case, a circle. They are learning the mathematical language to use when discussing circles. The students are developing meaning for the terms diameter, radii, intersection, and chord as they relate to the circle. In this context, the children have been learning vocabulary while being involved in solving what for them has become a real problem--determining the center of a circle. From examples of their thinking about this problem, Mrs. Clark can see that her students are beginning to realize that diameters must be congruent and that the radius is half of the diameter. She did not tell them how to find the center of a circle, but helped students construct the process.

Mrs. Clark reinforced through oral language and written symbols that mathematics is a discipline that has its own unique vocabulary and symbols. By learning to speak and write mathematically, students came to know mathematics. Lampert (1990) found in her research that students need to learn mathematical language as tools for communicating. Fennema, Carpenter, and Peterson (1989) noted that to help children make connections from concepts to procedures for writing symbols and algorithms, teachers must introduce ways to represent the knowledge that children have already acquired. The knowledge that Mrs. Clark helped the students construct is important for them in understanding that mathematics is about the relationships of procedures to concepts.

Conclusions and Implications

Thinking mathematically is not something that can be scheduled during a particular part of the day separate from the rest of the school day. Mrs. Clark consistently involved students in high-level discussions. She seized the moment during lessons when a student’s contribution had the potential to lead to rich discussions. Green, Kantor, and Rogers (1991) characterized these events as teachable moments. Mrs. Clark pushed students to think way ahead of what they already
knew. They learned to describe their thinking and became excited about mathematics. They built meaning through their actions and their interactions.

Many of the problems that were described in this article are what Lampert (1990) might call *good problems*. Lampert suggested that a good problem involves getting students to reveal and examine their assumptions about mathematics. With good problems, Mrs. Clark helped her students construct a way of thinking about mathematics that was rich with powerful conceptual connections. She communicated to students that mathematics makes sense. They saw that much of mathematics is learned by making connections with physical objects in the physical world. By reflecting upon their actions, they learned to generalize from the specifics of one problem to other problems. Mrs. Clark helped students build knowledge upon what they already knew and what made sense to them. Polya (1981) quoted Lichtenberg in saying, "What you have been obligated to discover by yourself leaves a path in your mind which you can use again when the need arises" (p. 103).

Mathematics textbooks should not contain page after page of procedural computations but should contain significant problems central to primary concepts in mathematics. One of the most significant factors in helping students to learn to think and value mathematics is the tasks or activities in which they are involved. Teachers should involve students in activities that encourage them to construct their thinking at deep levels. Teachers should ask students to reason and use their reasoning to build theories that they could prove. Students should be engaged cooperatively and individually in exploratory lessons involving hands-on tasks. These activities should be open-ended, challenging, and problematic for students. Teachers should find mathematical problems in everyday experience that are relevant for students. Teachers should use a few significant problems each day to teach the lesson. These problems should focus on the primary concepts that teachers want students to learn and to relate to their previous learning.

This article was about one teacher, Mrs. Clark. Mrs. Clark’s approach to teaching is complex and rich and provides an example that will challenge many teachers’ previous beliefs.
about how mathematics should be learned and taught. The problems that Mrs. Clark chose, the questions that she asked, the choices of manipulative materials, all sent messages to students about what she considered important for them to know about mathematics. Teachers need to share their own experiences with implementing constructivist ideas. Teachers need to talk about their teaching. This sharing can help others predict and better understand the experiences they might encounter. It is important to document how other teachers have constructed their own definitions of constructivist teaching. There is clearly not one way. This model of one teacher is meant to increase our understanding of what happens when a teacher interprets and implements a constructivist approach to mathematics learning and teaching.

REFERENCES


