1. For each of the following equations, state what type of conic section is describes. Then choose one equation to graph. Be sure to label all pertinent information (vertices, foci, etc...).

a) \(4x - 3y^2 = 1\)

After some algebra, this can be rewritten

\[
4 \cdot \frac{1}{3}(x - 1/12) = y^2, 
\]

which gives a parabola with vertex \((1/12, 0)\), focus \((1/12 + 1/3, 0)\) and directrix \(x = 1/12 - 1/3\).

b) \(4x^2 - 3y^2 = 1\)

After some algebra, this can be rewritten

\[
\frac{x^2}{(1/2)^2} - \frac{y^2}{(1/\sqrt{3})^2} = 1, 
\]

which gives a hyperbola centered at the origin with vertices \((\pm 1/2, 0)\), foci \((\pm \sqrt{7/12}, 0)\) and asymptotes \(y = \pm \sqrt{4/3}x\).

c) \(4x - 3y^2 = y\)

After some algebra, this can be rewritten

\[
4 \cdot \frac{1}{3}(x + 1/48) = (y + 1/6)^2, 
\]

which gives another parabola. This one has vertex \((-1/48, -1/6)\), focus \((1/3 - 1/48, -1/6)\) and directrix \(x = -1/3 - 1/48\).

d) \(2x - 4y^2 = x^2\)

After some algebra, this can be rewritten

\[
\frac{(x - 1)^2}{1^2} + \frac{y^2}{(1/2)^2} = 1, 
\]

which gives an ellipse with center \((1, 0)\), vertices \((1 \pm 1, 0)\), and foci \((1 \pm \sqrt{3/4}, 0)\).
e) Sorry, no graph here. I’ve done a lot more work (about 4 times as much) than necessary here. Hopefully, you could recognize which conic section you get without going through the algebra, which is really only needed if you want to get a good graph.

2. Sketch the graph of the polar curve \( r = 2 \cos \theta \) plotting at least six points by hand. Also, set up, but do not evaluate, an integral for the area enclosed by this curve.

Hopefully you see from your graph that this is a circle of radius 1 centered at the point \((1, 0)\) (which, oddly enough, is the same in either rectangular or polar coordinates). The arclength is just the circumference, \(2\pi\). But if you prefer, you can set it up via integrals. Using \(x = r \cos \theta\) and \(y = r \sin \theta\), you get \(dx/d\theta = -4 \sin \theta \cos \theta\) and \(dy/d\theta = 2 \cos^2 \theta - 2 \sin^2 \theta\). The curve is completely traversed from \(\theta = 0\) to \(\theta = \pi\), so the arclength is

\[
\int_{0}^{\pi} \sqrt{(dx/d\theta)^2 + (dy/d\theta)^2} \, d\theta
= \int_{0}^{\pi} \sqrt{16 \cos^2 \theta \sin^2 \theta + 4 \cos^4 \theta - 8 \cos^2 \theta \sin^2 \theta + 4 \sin^4 \theta} \, d\theta
= \int_{0}^{\pi} \sqrt{4 \cos^4 \theta + 8 \cos^2 \theta \sin^2 \theta + 4 \sin^4 \theta} \, d\theta
= \int_{0}^{\pi} \sqrt{(2 \cos^2 \theta + 2 \sin^2 \theta)^2} \, d\theta
= \int_{0}^{\pi} 2 \, d\theta = 2\pi.
\]
3. Find an equation for the plane through the points \( P(-1, 2, 3), Q(2, -3, 1) \) and \( R(3, 1, -2) \).

The vector from \( P \) to \( Q \) is \( <3, -5, -2> \) and the vector from \( P \) to \( R \) is \( <4, -1, -5> \). The cross product, which is \( <23, 7, 17> \), is a normal vector to the plane. The plane is given by

\[
23(x + 1) + 7(y - 2) + 17(z - 3) = 0
\]

(using the point \( P \)).

4. Find all points on the parametric curve \( x = e^t - t, \ y = t + t^2 \) where the tangent line is vertical. Find all points where the tangent line is horizontal.

The slope is given by

\[
\frac{dy}{dt} = \frac{1 + 2t}{dx/dt} = \frac{1 + 2t}{e^t - 1}.
\]

The slope is zero (the tangent line is horizontal) when \( t = -1/2 \), which gives the point \((e^{-1/2} + 1/2, -1/2 + (1/2)^2)\). The slope is undefined (the tangent line is vertical) when \( t = 0 \), which gives the point \((1, 0)\).

5. Identify the surface given in spherical coordinates by the equation \( \phi = \pi/4 \). (Be sure to give some explanation.) For extra credit, write this equation in either cylindrical or rectangular coordinates.

This surface is two right circular cones which open along the \( z \)-axis. The cones meet at the origin. If \( \phi = \pi/4 \) then \( z = \rho/\sqrt{2}, \ x = (\rho/\sqrt{2})\cos \theta \) and \( y = (\rho/\sqrt{2})\sin \theta \). In rectangular coordinates, you get \( x^2 + y^2 = z^2 \); in cylindrical, \( r^2 = z^2 \).