1. Find the velocity, speed and acceleration of the particle with position given by \( \mathbf{r}(t) = t^2 \mathbf{i} - \sin(t) \mathbf{j} + \tan^{-1} t \mathbf{k} \).

The velocity is the derivative of position,

\[
\mathbf{r}'(t) = 2t \mathbf{i} - \cos t \mathbf{j} + \frac{1}{1 + t^2} \mathbf{k}.
\]

The speed is the magnitude of velocity,

\[
|\mathbf{r}'(t)| = \sqrt{(2t)^2 + (\cos t)^2 + (1 + t^2)^{-2}}.
\]

The acceleration is the derivative of velocity,

\[
\mathbf{r}''(t) = 2 \mathbf{i} + \sin t \mathbf{j} - 2t(1 + t^2)^{-2} \mathbf{k}.
\]

2. Let \( w = xyz - \sin(x + z) + \cos y \), where \( x = s \cos t \), \( y = \sin t + s \) and \( z = e^{st} \). Use the chain rule to find \( \partial w/\partial s \) and \( \partial w/\partial t \).

Find the two total derivatives (matrices of partial derivatives) first. One is

\[
\begin{pmatrix}
\frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z}
\end{pmatrix} = 
\begin{pmatrix}
yz\cos(x+z) & xz\sin y & xy\cos(x+z)
\end{pmatrix}
\]

and the other is

\[
\begin{pmatrix}
\frac{\partial w}{\partial s} & \frac{\partial w}{\partial t}
\end{pmatrix} = 
\begin{pmatrix}
\cos t & -s \sin t & 1 & \cos t & te^{st} & se^{st}
\end{pmatrix}.
\]

By the chain rule, \( \frac{\partial w}{\partial s} \frac{\partial w}{\partial t} \) is the product of these two matrices, which is

\[
\begin{pmatrix}
yz\cos(x+z) & xz\sin y & xy\cos(x+z)
\end{pmatrix} \begin{pmatrix}
\cos t & -s \sin t & 1 & \cos t & te^{st} & se^{st}
\end{pmatrix}
\]

3. Evaluate the following limits.
a) \[ \lim_{(x,y) \to (2,3)} \frac{x^2 + y^2}{x^2 - y^2} \]

Just “plug in” \( x = 2 \) and \( y = 3 \) for this one; the limit is \(-\frac{13}{5}\).

b) \[ \lim_{(x,y) \to (0,0)} \frac{x^2 - xy + y^2}{x^2 + y^2} \]

Convert to polar coordinates. This limit is

\[ \lim_{r \to 0^+} \frac{r^2 \cos^2 \theta - r^2 \cos \theta \sin \theta + r^2 \sin^2 \theta}{r^2} = \lim_{r \to 0^+} 1 - \cos \theta \sin \theta. \]

Since this depends on \( \theta \), the original limit does not exist.

4. Let \( f(x, y) = xye^x \). Find the directional derivative of \( f \) at the point \((1, 2)\) in the direction of \(<2,1>\).

The directional derivative is the dot product of the gradient with the direction. First,

\[ \nabla f = \langle f_x, \ f_y \rangle = \langle ye^x + xye^x, \ xe^x \rangle. \]

So

\[ \nabla f(1, 2) = \langle 4e, \ e \rangle. \]

Next, the direction of \(<1,2>\) is \(\sqrt{5}^{-1} <1,2>\). So the directional derivative is \(\sqrt{5}^{-1}6e\).

5. Find the absolute maximum and minimum values of \( x^2 - 2xy + y \) on the rectangle \( R = \{(x, y): \ -1 \leq x \leq 1, \ -2 \leq y \leq 2\} \).

The first step is to find the critical points inside the rectangle. To do that, you solve \( f_x = f_y = 0 \). In this case, we get \( 2x - 2y = -2x + 1 = 0 \), and the only solution is \((1/2, 1/2)\) (which is in the rectangle). At this point, the function value is \(1/4\).

Next, we parametrize the boundary of the rectangle. This is best done in four pieces. The top of the rectangle is given by

\[ x = t, \ y = 2, \ -1 \leq t \leq 1. \]

The right hand side is given by

\[ x = 1, \ y = t, \ -2 \leq t \leq 2. \]
The bottom is given by
\[ x = t, \ y = -2, \ -1 \leq t \leq 1. \]
The left hand side is given by
\[ x = -1, \ y = t, \ -2 \leq t \leq 2. \]
On the top of the rectangle, the function \( f \) is given by
\[ f(t, 2) = t^2 - 4t + 2, \ -1 \leq t \leq 1, \]
which (as you can check) has an absolute maximum of \( f(-1, 2) = 7 \) and an absolute minimum of \( f(1, 2) = -1 \).
On the right hand side of the rectangle, the function \( f \) is given by
\[ f(1, t) = 1 - t, \ -2 \leq t \leq 2, \]
which has an absolute maximum of \( f(1, -2) = 3 \) and an absolute minimum of \( f(1, 2) = -1 \).
On the bottom of the rectangle, the function \( f \) is given by
\[ f(t, -2) = t^2 + 4t - 2, \ -1 \leq t \leq 1, \]
which has an absolute maximum of \( f(1, -2) = 3 \) and an absolute minimum of \( f(-1, -2) = -5 \).
Finally, on the left hand side of the rectangle, the function \( f \) is given by
\[ f(-1, t) = 1 + 3t, \ -2 \leq t \leq 2, \]
which has an absolute maximum of \( f(-1, 2) = 7 \) and an absolute minimum of \( f(-1, -2) = -5 \).
The absolute maximum of \( f \) on the rectangle is \( f(-1, 2) = 7 \) and the absolute minimum is \( f(-1, -2) = -5 \).

6. Find the linearization of \( f(x, y) = e^{x+y} - \ln(x^2 + y^2) \) at the point \((0,1)\) and use this to approximate \( f(1, 9) \).

The linearization of \( f \) at \((0,1)\) is
\[ L(x, y) = f_x(0,1)(x - 0) + f_y(y - 1) + f(0,1). \]
The partial derivatives are

\[ f_x = e^{x+y} - \frac{2x}{x^2 + y^2} \quad \text{and} \quad f_y = e^{x+y} - \frac{2y}{x^2 + y^2}. \]

The function value is

\[ f(0, 1) = e. \]

Plugging \( x = 0 \) and \( y = 1 \) into the partial derivatives, you get

\[ L(x, y) = e(x - 0) + (e - 2)(y - 1) + e. \]

Finally, \( f(.1, .9) \) is approximately

\[ L(.1, .9) = e(.1) + (e - 2)(-.1) + e. \]