Axioms for Polynomials

The set of polynomials with rational coefficients, $\mathbb{Q}[X]$, is a non-empty set with a binary operations $+$ and $\times$. It has a shockingly familiar set of axioms.

The addition $+$ satisfies the following:

1. $(a + b) + c = a + (b + c)$ for any $a, b, c \in \mathbb{Q}[X]$ ($+$ is associative);
2. there is a polynomial $0$ where $0 + a = a + 0 = a$ for all $a \in \mathbb{Q}[X]$ (there is an additive identity);
3. for every $a \in \mathbb{Q}[X]$ there is a $b \in \mathbb{Q}[X]$ where $a + b = b + a = 0$ (every element has an additive inverse); and
4. $a + b = b + a$ for all $a, b \in \mathbb{Q}[X]$ (addition is commutative).

The multiplication $\times$ satisfies the following:

1. it is associative;
2. it has an identity not equal to the additive identity;
3. it distributes through addition on both the left and right, i.e., $a \times (b + c) = a \times b + a \times c$ and $(a + b) \times c = a \times c + b \times c$ for all $a, b, c \in \mathbb{Q}[X]$; and
4. it is commutative.

Is there is an order relation on $\mathbb{Q}[X]$ which totally orders $\mathbb{Q}[X]$ in the same way $\leq$ totally orders $\mathbb{Z}$?