Here are three more examples of groups.

1) The set of even integers (usually denoted $2\mathbb{Z}$) with addition.
(Please note that this is most definitely not $\mathbb{Z}_2$.)

2) The set of even permutations in $S_4$ with composition.
More generally, the set of even permutations in $S_n$ for any $n$.
(These groups are usually denoted $A_n$ and are called the alternating group on $n$ letters.)

3) The set of upper triangular and invertible $2 \times 2$ matrices with matrix multiplication.

In these examples, did we need to check for associativity?

Is there any way to make your life easier when you want to check if a particular subset of a known group is a group?
**Definition:** A *subgroup* of a group $G$ is a subset of $G$ which is a group in its own right, using the same binary operation.

Is it possible for a subgroup of a group to have a new and different identity element? In other words, if $G$ is a group with identity element $e$ and $H$ is a subgroup of $G$, then $H$ has an identity element, too. Must the identity element of $H$ be $e$?

Generally speaking, in order for a subset $H$ of a group $G$ to be a subgroup, we must be sure that

- $H$ is closed: if $a$ and $b$ are elements of $H$, then so is $ab$.
- The identity element $e$ of $G$ is in $H$.
- For every element $a \in H$, $a^{-1}$ is an element of $H$, too.

**Corollary 3.2.3:** Let $G$ be a group and $H$ be a non-empty subset of $G$. Then $H$ is a subgroup if and only if $ab^{-1} \in H$ whenever $a, b \in H$. 