Math 229 Section 1 Quiz #7 Solutions

1. Show that the equation \(2x - 1 - \sin x = 0\) has exactly one real root.

The function \(f(x) = 2x - 1 - \sin x\) is continuous. Since \(f(0) = -1\) and \(f(2) = 3 - \sin 2 \geq 3 - 2 = 1\), the Intermediate Value Theorem implies that \(f(c) = 0\) for some \(c\) in the interval \((0, 2)\). Moreover,

\[
f'(x) = \frac{d^2x}{dx} - \frac{d1}{dx} - \frac{d\sin x}{dx} = 2 - \cos x \geq 2 - 1 = 1,
\]

so Rolle’s Theorem implies that \(f(x)\) cannot equal zero more than once. Thus, \(f(x)\) is equal to zero exactly once.

2. Find the critical numbers of \(f(x) = x^4(x - 1)^3\). What does the First Derivative Test tell you?

By the product rule and the chain rule with \(u = x - 1\),

\[
f'(x) = \frac{d^4x}{dx}(x - 1)^3 + x^4\frac{d(x - 1)^3}{dx} = 4x^3(x - 1)^3 + x^4\frac{du^3}{du}\frac{dx}{du} = 4x^3(x - 1)^3 + x^43u^2\left(\frac{dx}{dx} - \frac{d1}{dx}\right) = 4x^3(x - 1)^3 + 3x^4(x - 1)^2 = x^3(x - 1)^2(4(x - 1) + 3x) = x^3(x - 1)^2(7x - 4).
\]

The critical numbers are 0, 1 and 4/7. We also see that \(f'\) is positive on \((-\infty, 0)\) and \((4/7, \infty)\); it is negative on \((0, 1)\) and \((1, 4/7)\). The First Derivative Test tells us that we have a local maximum at \(x = 0\) and a local minimum at \(x = 4/7\).

3. For \(g(x) = 200 + 8x^3 + x^4\), find the intervals of increase or decrease, the local maximum and minimum values, the intervals of concavity and the inflection points.

First find \(g'(x)\) and \(g''(x)\):

\[
g'(x) = \frac{d200}{dx} + \frac{d8x^3}{dx} + \frac{dx^4}{dx} = 8\frac{dx^3}{dx} + 4x^3 = 24x^2 + 4x^3,
\]

and

\[
g''(x) = \frac{d24x^2}{dx} + \frac{d4x^3}{dx} = 24\frac{dx^2}{dx} + 4\frac{dx^3}{dx} = 48x + 12x^2.
\]
Factoring both of these gives

\[ g'(x) = 4x^2(6 + x), \quad g''(x) = 12x(4 + x). \]

You can check that \( g' \) is negative on \((-\infty, -6)\) and positive on \((-6, 0)\), and \((0, \infty)\). We have a local minimum at \(-6\) and just a horizontal tangent with no local extremum at 0. You can check that \( g'' \) is negative on \((-4, 0)\) and positive on \((-\infty, -4)\) and \((0, \infty)\). The graph is concave down on \((-4, 0)\) and concave up on \((-\infty, -4)\) and \((0, \infty)\); there are points of inflection at 0 and \(-4\).