Math 620 Fall 2012
Week 3: Group Actions

Major Definitions: $G$-set, orbit, stabilizer, conjugation, normalizer, centralizer, $p$-group

Major Theorems: the “stabilizer/orbit theorem”, the orbit formula, the class equation, Cayley’s Theorem

Exercises:
1. Let $X_n$ be the set of vertices of the regular $n$-gon, so that $D_n$ acts as a transitive permutation group on $X_n$. For each $x \in X_n$, show that the stabilizer of $x$ is a group of order 2.
2. If $n$ is odd, show that $D_n$ has $n$ conjugate subgroups of order 2.
3. Show that $D_n$ has a trivial center when $n$ is odd. What if $n$ is even?
4. If $G$ is a $p$-group, show that $G$ has a non-trivial center.
5. Prove Cauchy’s Theorem: if $G$ is a finite group and $p$ is a prime dividing the order of $G$, then $G$ has an element of order $p$. I’m well aware that the text has a proof. However, I want you to use induction on the order of $G$ and the class equation to reduce this to the case where $G$ is abelian.
6. If $G$ is a finite $p$-group and $N$ is a non-trivial normal subgroup of $G$, then show that $N$ has a non-trivial intersection with the center of $G$.
7. Do page 51 #4.
8. If $p$ is a prime and $\tau \in S_p$ is a transposition, show that $\tau$ and $(1, 2, \ldots, p)$ generate $S_p$.
9. If $p$ is a prime and $G$ is a transitive subgroup of $S_p$ that contains a transposition, show that $G = S_p$.

Numbers 3, 5, and 9 are to be presented in class on Friday.