Major Definitions: Sylow $p$-subgroup, simple group

Major Theorems: the three Sylow theorems

Exercises:
1. Show that a group of order 108 is not simple.
2. If $G$ is a group of order $pq$, where $p$ and $q$ are distinct primes, then $G$ is not simple.
3. If $G$ is an infinite group and $H$ is a subgroup of finite index, then $H$ contains a normal subgroup (of $G$) of finite index.
4. Suppose $G$ is a finite group and $p$ is the smallest prime dividing the order of $G$. Suppose further that $H$ is a subgroup of index $p$. Show that $H$ is a normal subgroup.
5. If $G$ is a group of order $p^n$, where $p$ is a prime and $n > 0$, and $0 \leq k \leq n$, then $G$ has a normal subgroup of order $p^k$. 