7.7 Exercises

1. Let \( I = \int_0^4 f(x) \, dx \), where \( f \) is the function whose graph is shown.
   (a) Use the graph to find \( L_4 \), \( R_4 \), and \( M_4 \).
   (b) Are these underestimates or overestimates of \( I \)?
   (c) Use the graph to find \( T_4 \). How does it compare with \( I \)?
   (d) For any value of \( n \), list the numbers \( L_n \), \( R_n \), \( M_n \), \( T_n \), and \( I \) in increasing order.

2. The left, right, Trapezoidal, and Midpoint Rule approximations were used to estimate \( \int_1^4 f(x) \, dx \), where \( f \) is the function whose graph is shown. The estimates were 0.7811, 0.8675, 0.8632, and 0.9540, and the same number of subintervals were used in each case.
   (a) Which rule produced the estimate?
   (b) Between which two approximations does the true value of \( \int_1^4 f(x) \, dx \) lie?

3. Estimate \( \int_0^1 \cos(x^2) \, dx \) using (a) the Trapezoidal Rule and (b) the Midpoint Rule, each with \( n = 4 \). From a graph of the integrand, decide whether your answers are underestimates or overestimates. What can you conclude about the true value of the integral?

4. Draw the graph of \( f(x) = \sin(\frac{1}{2}x^2) \) in the viewing rectangle \([0, 1] \times [0, 0.5]\) and let \( I = \int_0^1 f(x) \, dx \).
   (a) Use the graph to decide whether \( L_2 \), \( R_2 \), \( M_2 \), and \( T_2 \) underestimate or overestimate \( I \).
   (b) For any value of \( n \), list the numbers \( L_n \), \( R_n \), \( M_n \), \( T_n \), and \( I \) in increasing order.

(c) Compute \( L_5 \), \( R_5 \), \( M_5 \), and \( T_5 \). From the graph, which do you think gives the best estimate of \( I \)?

5–6 Use (a) the Midpoint Rule and (b) Simpson’s Rule to approximate the given integral with the specified value of \( n \). (Round your answers to six decimal places.) Compare your results to the actual value to determine the error in each approximation.

5. \( \int_0^\pi \frac{x}{1 + x^2} \, dx \), \( n = 10 \)

6. \( \int_0^\pi \cos x \, dx \), \( n = 4 \)

7–18 Use (a) the Trapezoidal Rule, (b) the Midpoint Rule, and (c) Simpson’s Rule to approximate the given integral with the specified value of \( n \). (Round your answers to six decimal places.)

7. \( \int_1^4 \sqrt{x^3 - 1} \, dx \), \( n = 10 \)

8. \( \int_0^1 \frac{1}{1 + x^6} \, dx \), \( n = 8 \)

9. \( \int_0^2 \frac{e^x}{1 + x^2} \, dx \), \( n = 10 \)

10. \( \int_0^{e/2} \frac{\sqrt{1 + \cos x}}{1 + x^2} \, dx \), \( n = 4 \)

11. \( \int_1^3 \sqrt{\ln x} \, dx \), \( n = 6 \)

12. \( \int_0^1 \sin(x^3) \, dx \), \( n = 10 \)

13. \( \int_0^1 e^{-\theta} \sin \theta \, d\theta \), \( n = 8 \)

14. \( \int_0^1 \sqrt{x} e^{-\theta} \, d\theta \), \( n = 10 \)

15. \( \int_0^1 \frac{\cos x}{x} \, dx \), \( n = 8 \)

16. \( \int_0^1 \frac{\sin x^3}{x} \, dx \), \( n = 10 \)

17. \( \int_0^1 e^{\theta^2} \, d\theta \), \( n = 10 \)

18. \( \int_0^1 \cos x^2 \, dx \), \( n = 10 \)

19. (a) Find the approximations \( T_n \) and \( M_n \) for the integral \( \int_0^1 \cos(x^2) \, dx \).
   (b) Estimate the errors in the approximations of part (a).
   (c) How large do we have to choose \( n \) so that the approximations \( T_n \) and \( M_n \) to the integral in part (a) are accurate to within 0.0001?

20. (a) Find the approximations \( T_{10} \) and \( M_{10} \) for \( \int_0^1 e^{\theta^2} \, d\theta \).
   (b) Estimate the errors in the approximations of part (a).
   (c) How large do we have to choose \( n \) so that the approximations \( T_n \) and \( M_n \) to the integral in part (a) are accurate to within 0.0001?

21. (a) Find the approximations \( T_{10} \), \( M_{10} \), and \( S_{10} \) for \( \int_0^\pi \sin x \, dx \) and the corresponding errors \( E_T \), \( E_M \), and \( E_S \).