1. Nutrition Problem
Sarah decides to make rice and soybeans part of her staple diet. The object is to design a lowest–cost diet that provides certain minimum levels of protein, calories, and vitamin $B_{12}$ (riboflavin).
One cup of uncooked rice costs 21 cents and contains 15 grams of protein, 810 calories, and $\frac{1}{3}$ mg of riboflavin.
One cup of uncooked soybeans costs 14 cents and contains 22.5 grams of protein, 270 calories, and $\frac{1}{3}$ mg of riboflavin.
The minimum daily requirements are: 90 grams of protein, 1620 calories, and 1 mg of $B_{12}$.
Design the lowest cost diet that meets these requirements.

2. Set the Variables
Let

- $x =$ the number of cups of uncooked rice in her diet
- $y =$ the number of cups of uncooked soybeans in her diet

The problem is to find the values of $x$ and $y$ which will

- minimize the cost and
- provide the daily requirements of protein, calories, and riboflavin.

3. Organizing the Data

<table>
<thead>
<tr>
<th>Category</th>
<th>Rice</th>
<th>Soybeans</th>
<th>Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Protein</td>
<td>15</td>
<td>22.5</td>
<td>90</td>
</tr>
<tr>
<td>Calories</td>
<td>810</td>
<td>270</td>
<td>1620</td>
</tr>
<tr>
<td>Riboflavin</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
<td>1</td>
</tr>
<tr>
<td>Cost</td>
<td>21</td>
<td>14</td>
<td>$C$</td>
</tr>
</tbody>
</table>

Nutrition Inequalities:
- Protein: $15x + 22.5y \geq 90$
- Calories: $810x + 270y \geq 1620$
- Riboflavin: $\frac{1}{3}x + \frac{1}{3}y \geq 1$

Cost Equation:
- Cost: $C = 21x + 14y$

4. Simplify the Inequalities
Nutrition Inequalities:
(1) Protein: $15x + 22.5y \geq 90$
(2) Calories: $810x + 270y \geq 1620$
(3) Riboflavin: $\frac{1}{3}x + \frac{1}{3}y \geq 1$

To remove the decimal 22.5 in (1), multiply (1) by 2:
(1') Protein: $30x + 45y \geq 180$
Now notice that 30, 45, and 180 are all divisible by 15. So divide (1') by 15:
(1") Protein: $2x + 3y \geq 12$

In (2) notice that 810, 270, and 1620 are all divisible by 270. So divide (2) by 270:
(2') Calories: $3x + y \geq 6$

To remove the denominators in (3), multiply (3) by 9:
(3') Riboflavin: $x + 3y \geq 9$
5. Nutrition Words → Math

Minimize the cost \( C = 21x + 14y \)
subject to the constraints
- \( 2x + 3y \geq 12 \)
- \( 3x + y \geq 6 \)
- \( x + 3y \geq 9 \)
- \( x \geq 0, \ y \geq 0 \)

6. Protein Halfplane

Halfplane: \( 2x + 3y \geq 12 \)
Line: \( 2x + 3y = 12 \)
\( x \)-intercept: \((6, 0)\)
\( y \)-intercept: \((0, 4)\)
Test point equation: \( 0 \geq 12 \)
Position: above line

7. Calorie Halfplane

Halfplane: \( 3x + y \geq 6 \)
Line: \( 3x + y = 6 \)
\( x \)-intercept: \((2, 0)\)
\( y \)-intercept: \((0, 6)\)
Test point equation: \( 0 \geq 6 \)
Position: above line

8. Riboflavin Halfplane

Halfplane: \( x + 3y \geq 9 \)
Line: \( x + 3y = 9 \)
\( x \)-intercept: \((9, 0)\)
\( y \)-intercept: \((0, 3)\)
Test point equation: \( 0 \geq 9 \)
Position: above line
9. Where do Protein and Calorie Lines Intersect?

- Write the equations:
  - Protein \(2x + 3y = 12\)
  - Calorie \(3x + y = 6\)
- Multiply Calorie equation by 3
  - Calorie \(9x + 3y = 18\)
  - Protein \(2x + 3y = 12\)
- Subtract \(7x = 6\)
- So \(x = \frac{6}{7}\)
- Plug \(x = \frac{6}{7}\) back into the Calorie equation
  - \(y = 6 - 3x = 6 - 3 \cdot \frac{6}{7} = \frac{42}{7} - \frac{18}{7} = \frac{24}{7}\)
- The intersection point is \(\left(\frac{6}{7}, \frac{24}{7}\right)\)

10. Where do Protein and Riboflavin Lines Intersect?

- Write the equations:
  - Protein \(2x + 3y = 12\)
  - Ribofl \(x + 3y = 9\)
- Multiply Calorie equation by 3
  - Calorie \(9x + 3y = 18\)
  - Ribofl \(x + 3y = 9\)
- Subtract \(8x = 9\)
- So \(x = \frac{9}{8}\)
- Plug \(x = \frac{9}{8}\) back into the Riboflavin equation
  - \(y = 9 - x = 9 - \frac{9}{3} = 2\)
- The intersection point is \((3, 2)\)

11. Where do Calorie and Riboflavin Lines Intersect?

- Write the equations:
  - Calorie \(3x + y = 6\)
  - Ribofl \(x + 3y = 9\)
- Multiply Calorie equation by 3
  - Calorie \(9x + 3y = 18\)
  - Ribofl \(x + 3y = 9\)
- Subtract \(8x = 9\)
- So \(x = \frac{9}{8}\)
- Plug \(x = \frac{9}{8}\) back into the Calorie equation
  - \(y = 6 - 3x = 6 - 3 \cdot \frac{9}{8} = \frac{48}{8} - \frac{27}{8} = \frac{21}{8}\)
- The intersection point is \(\left(\frac{9}{8}, \frac{21}{8}\right)\)

12. Feasible Region
13. Solving the Nutrition Problem

The list of corner points is:
(0, 6), (\(\frac{6}{7}, \frac{24}{7}\)), (3, 2), (9, 0)

The intersection \(\left(\frac{9}{8}, \frac{21}{8}\right)\) of Lines 2 and 3 is not a corner point because it is not on halfplane
\[2x + 3y \geq 12:\]
\[2 \cdot \frac{9}{8} + 3 \cdot \frac{21}{8} = \frac{18}{8} + \frac{63}{8} = \frac{81}{8} = 10.125 < 12\]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>C  = 21x + 14y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
<td>84</td>
</tr>
<tr>
<td>(\frac{6}{7})</td>
<td>(\frac{24}{7})</td>
<td>18 + 48 = 66</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>91</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>189</td>
</tr>
</tbody>
</table>

Minimum value of \(C\) is 66 at the point \(\left(\frac{6}{7}, \frac{24}{7}\right)\)

15. Solution: Determine Variables

At first it appears that 4 variables will be needed, since there are two stores and two warehouses, hence 4 possible shipping combinations.

A closer look shows that only two variables \(x\) and \(y\) are needed.

For if \(x\) represents the number of TV sets to be shipped from DeKalb to Schaumburg, then since Schaumburg needs 25 sets, the number of TV sets to be shipped from Elkhart to Schaumburg is \(25 - x\).

Similarly, if \(y\) represents the number of TV sets to be shipped from DeKalb to Aurora, then since Aurora needs 30 sets, the number of TV sets to be shipped from Elkhart to Aurora is \(30 - y\).

14. Transportation Problem

A Chicago TV dealer has stores in Schaumburg and Aurora and warehouses in DeKalb and Elkhart, Indiana. The cost of shipping a 48 inch flat screen TV set from DeKalb to Schaumburg is $6; from DeKalb to Aurora is $3; from Elkhart to Schaumburg, $9; and from Elkhart to Aurora is $5. Suppose the Schaumburg store orders 25 TV sets and the Aurora store orders 30. The DeKalb warehouse has a stock of 45 TV sets and the Elkhart warehouse has 40.

What is the most economical way to supply the required sets to the two stores?

16. Cost Table

<table>
<thead>
<tr>
<th>Warehouse–Store</th>
<th>Number</th>
<th>Cost per set</th>
</tr>
</thead>
<tbody>
<tr>
<td>DeKalb–Schaumburg</td>
<td>(x)</td>
<td>$6</td>
</tr>
<tr>
<td>DeKalb–Aurora</td>
<td>(y)</td>
<td>$3</td>
</tr>
<tr>
<td>Elkhart–Schaumburg</td>
<td>(25 - x)</td>
<td>$9</td>
</tr>
<tr>
<td>Elkhart–Aurora</td>
<td>(30 - y)</td>
<td>$5</td>
</tr>
</tbody>
</table>

Total Cost:
\[C = 6x + 3y + 9(25 - x) + 5(30 - y)\]
\[= 6x + 3y + 225 - 9x + 150 - 5y\]
\[= -3x - 2y + 375\]
17. Constraints

There are two kinds of constraints:

- none of \( x, y, 25 - x, 30 - y \) can be negative
  Equivalently, \( x \geq 0, y \geq 0, x \leq 25, y \leq 30 \)
- a warehouse cannot ship more TVs than it has in stock.
  Since DeKalb ships \( x + y \) sets and has 45 in stock, we get the constraint:
    \( x + y \leq 45 \)
  Since Elkhart ships \((25 - x) + (30 - y)\) sets and has 40 in stock, we get the constraint:
    \((25 - x) + (30 - y) \leq 40\)
    or \(55 - x - y \leq 40\)
    or \(15 \leq x + y\)

18. Transportation Words → Math

Minimize the cost \( C = -3x - 2y + 375 \)
subject to the constraints

- \( x \leq 25, y \leq 30 \)
- \( 15 \leq x + y \)
- \( x + y \leq 45 \)
- \( x \geq 0, y \geq 0 \)

19. The Halfplanes

<table>
<thead>
<tr>
<th>Halfplane</th>
<th>Line</th>
<th>Intercept(s)</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 15 \leq x + y )</td>
<td>( 15 = x + y )</td>
<td>(15, 0) (0, 15)</td>
<td>above</td>
</tr>
<tr>
<td>( x + y \leq 45 )</td>
<td>( x + y = 45 )</td>
<td>(45, 0) (0, 45)</td>
<td>below</td>
</tr>
<tr>
<td>( y \leq 30 )</td>
<td>( y = 30 )</td>
<td>(0, 30)</td>
<td>below</td>
</tr>
<tr>
<td>( x \leq 25 )</td>
<td>( x = 25 )</td>
<td>(25, 0)</td>
<td>left</td>
</tr>
</tbody>
</table>

The constraints \( x, y \geq 0 \) imply that the region lies in the First Quadrant.

20. Feasible Region

21. Solving the Transportation Problem

The list of corner points is:
\((0, 15), (0, 30), (15, 30), (25, 20), (25, 0)\) and \((15, 0)\)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( C = 375 - 3x - 2y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15</td>
<td>345</td>
</tr>
<tr>
<td>0</td>
<td>30</td>
<td>315</td>
</tr>
<tr>
<td>15</td>
<td>30</td>
<td>270</td>
</tr>
<tr>
<td>25</td>
<td>20</td>
<td>260</td>
</tr>
<tr>
<td>25</td>
<td>0</td>
<td>300</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>330</td>
</tr>
</tbody>
</table>

Minimum value of \( C \) is
- 260 and occurs at the point \((25, 20)\)